7. (20 pts) Two point masses \( m_1 \) and \( m_2 \) are joined by a spring having zero natural length and spring constant \( k \). They are thrown out at \( t = 0 \) into intergalactic space (no gravity) as shown below.

(a) Neglecting the gravitational interaction between the particles and their possible collision, write the Lagrangian describing their motion.

\[
L(r_1, r_2; \dot{r}_1, \dot{r}_2) = \frac{1}{2} m_1 \dot{\vec{r}_1}^2 + \frac{1}{2} m_2 \dot{\vec{r}_2}^2 - \frac{1}{2} k \left( \vec{r}_1 - \vec{r}_2 \right)^2
\]

(b) Define the center of mass coordinate \( \vec{R} \) and relative coordinate \( \vec{r} \).

\[
\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}
\]

\[
\vec{r} = \vec{r}_1 - \vec{r}_2
\]