Problems

1. Show, for the infinite well, that the average position \( \langle x \rangle \) is independent of the quantum state. *Hint: Use the integral formula: \( \int_0^{2n\pi} u \cos u \, du = 0 \) (for integer \( n \)).

2. Carefully sketch the wave function and the probability density for the \( n = 4 \) state of a particle in a finite potential well.

3. **SMM, Chapter 5, problem 26.** *Hint: Check Example 5.13 if you are stuck.*

4. Calculate \( \langle x \rangle, \langle x^2 \rangle, \) and \( \Delta x \) for a quantum oscillator in its ground state. *Hint 1: Is the integral, over all \( x \), of an odd function zero?*  
   *Hint 2: Use the integral formula \( \int_0^\infty u^2 e^{-au^2} \, du = \frac{2}{a} \sqrt{\pi} \) \( a > 0 \).*

5. **SMM, Chapter 6, problem 1.**

6. Sketch careful, qualitatively accurate plots for the stated wave functions in each of the potentials shown. *Important: Check that your wave function has the correct symmetry, number of nodes, relative wavelengths, maximum values of amplitudes and relative rate of decrease outside the well.*
   (a) The ground state, 1\(^{st}\) and 2\(^{nd}\) excited wave functions of the quantum oscillator. Realize that this corresponds to the 1\(^{st}\), 2\(^{nd}\) and 3\(^{rd}\) bound state.
   (b) The 5\(^{th}\) bound state of the finite square well with a two level floor.
   (c) The 5\(^{th}\) bound state of the finite square well with a ramped floor.