Problems

1. SMM, Chapter 1, Problem 26.

2. SMM, Chapter 1, Problem 27.

3. SMM, Chapter 1, Problem 33.

4. SMM, Chapter 1, Problem 36.

5. Follow up to Homework 2, problem 7. If we solve the scalar relativistic Newton’s equation, \( F = \frac{dp}{dt} = \frac{\partial}{\partial t}(\gamma mv) \), where \( \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \), for the velocity \( v(t) \), subject to the conditions of a constant force \( F_0 \) and initial velocity \( v_0 \), we get:

\[
v(t) = \left(\frac{F_0}{m} t + \frac{v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}\right) \left(1 + \frac{1}{c^2} \left(\frac{F_0}{m} t + \frac{v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}\right)^2\right)^{-\frac{1}{2}}
\]

Consider a small 1 kg object starting from rest, subject to a force \( F_0 \) of 100 N.
(a) Using the non-relativistic speed equation (i.e. \( v(t) = \frac{F_0}{m} t + v_0 \)), how long would it take for the object to reach the speed of light? Give your answer in days.
(b) Using the relativistic speed equation, what’s the speed after the same period of time?
(c) Plot both equations as a function of time from 0 to the time computed in (a)?

6. Extra Credit Problem 1. Derive the relativistic speed equation from problem 5. This looks a lot harder that it is. Give it a try.

7. Extra Credit Problem 2. Show that the relativistic speed equation reduces to the non-relativistic speed equation when \( v(t) \ll c \). Hint: To do this, we must assume that \( v_0 \ll c \) and that \( t \) is very small. Mathematically speaking, this is the equivalent of throwing out terms that have \( v_0^2/c^2 \) and \( t^2 \) in them.