3. According to the conservation of Momentum, we have:
\[ m \vec{v}_0 = \frac{m}{3} (\vec{u}_1 + \vec{u}_2 + \vec{u}_3) \]

\( \Rightarrow \) \[ X \text{ direction: } m \vec{v}_0 = \frac{m}{3} (\vec{v}_0 + \vec{v}_3 - \cos \theta_3 + \vec{v}_2 - \cos \theta_2) \]

\[ Y \text{ direction: } 0 = \frac{m}{3} (\vec{v}_3 \sin \theta_3 - \vec{v}_2 \sin \theta_2) \]

And, \[ \vec{v}_3 = \vec{v}_2 \] \[ \theta_2 + \theta_3 = \frac{\pi}{2} \] \[ \vec{v}_2 = \vec{v}_3 = \vec{v}_0 \]

Combining (3.1) in (3.4), we get: \[ \theta_2 = \theta_3 = \frac{\pi}{4}, \quad \vec{v}_2 = \vec{v}_3 = \vec{v}_0 \]

3.8. (a) Because \[ m \vec{v} = -m \vec{v}_E - mg = 0 \]

We have \[ m \vec{v}_E = -mg \Rightarrow \vec{v}_E = -\frac{mg}{m} \vec{v} \Rightarrow \vec{v}_E(t) = -\int_{t_0}^{t} \frac{mg}{m} \vec{v} \, dt = \vec{v}_E \left( \ln \frac{m_0}{m_0 - \frac{mg}{m} \vec{v}} \right) = \vec{v}_E \ln \left( \frac{m}{m - \frac{mg}{m} \vec{v}} \right) \]

(b) \[ \vec{v}_E = 3 \, \text{km/s}, \quad \lambda = 0.1 \quad \text{we have } \Delta t = 2.25 \]

13. \[ m \vec{v} = -m \vec{v}_E - mg \Rightarrow \vec{v} = -\frac{mg}{m} \vec{v}_E - g \Rightarrow \vec{v}(t) = -\int_{t_0}^{t} \frac{mg}{m} \vec{v} \, dt - gt = \vec{v}_E \left( \ln \frac{m_0}{m_0 - \frac{mg}{m} \vec{v}_E - gt} \right) \]

\[ \vec{y}(t) = \int_{t_0}^{t} \vec{v}_E \, dt = -\frac{1}{2} gt^2 + \int_{t_0}^{t} \vec{v}_E \ln \left( \frac{m_0}{m_0 - \frac{mg}{m} \vec{v}_E - gt} \right) \, dt \]

\[ = \int_{m_0}^{m} \ln \frac{m_0}{m} \, dm = m \ln \frac{m_0}{m} - m_0 \ln \frac{m_0}{m_0} - \int_{m_0}^{m} \ln \left( \frac{m_0}{m} + 1 \right) \, dm = m \ln \frac{m_0}{m} + m - m_0 \]

\[ \Rightarrow \vec{y}(t) = -\frac{1}{2} gt^2 + \frac{\vec{v}_E}{c} \left( m \ln \frac{m_0}{m} - kt \right) = \vec{v}_E t - \frac{1}{2} gt^2 - \frac{\vec{v}_E}{c} \left( \ln \frac{m_0}{m} \right) \]

For \[ \vec{v}_E = 3 \, \text{km/s}, \quad m_0 = 2 \times 10^6 \text{kg}, \quad m = 1 \times 10^6 \text{kg}, \quad t = 120 \text{ s} \]

we have \[ k = \frac{x_0}{t_0} \text{ km/s} \]

\[ y = 3.99 \times 10^4 \text{ m} \]

16. \[ \vec{x}_{CM} = \frac{m_S \vec{x}_S + m_E \vec{x}_E}{m_S + m_E} \Rightarrow \vec{x}_E = \frac{6.0 \times 10^{14}}{2.0 \times 10^{10}} \times 1.5 \times 10^8 \text{ km} = 4.5 \text{ km} \]

So \[ \vec{x}_{CM} \text{ is close to the center of the Sun.} \]
According to the symmetry, we have \( X_{cm} = 0 \)
\[
Y_{cm} = \frac{\int y \, dA}{M} = \frac{\int \rho \sin \phi \, r^2 \, d\phi \, dr \, ds}{\pi R^2} = \frac{\rho \sin \phi}{3 \pi}
\]

So, the position of the CM is \((0, \frac{\rho \sin \phi}{3 \pi})\)

Note:
1. We could always choose \(xy\) in the plane of the metal, which is 2D problem.
2. We could not directly use polar coordinates to determine \(cm\), for \(\phi\) and \(r\) are not well-defined direction.

According to the symmetry, we could easily have:
\[
X_{cm} = Y_{cm} = 0
\]
\[
Z_{cm} = \frac{\int \rho \, r \, dV}{M} = \frac{\int \rho \, r \, \sin \phi \, r^2 \, \cos \phi \, \sin \phi \, d\phi \, dr \, ds}{\frac{2}{3} \pi R^3} = \frac{3}{8} R
\]
\[
\hat{r}_{cm} = (0, 0, \frac{3}{8} R)
\]

3.25. Because there is no torque about the origin of the circle, we have:
\[
\ell = \int r \times \vec{\tau} = m \hat{r} \times \int (\hat{\omega} \times \vec{r}) = m \hat{r} \times \int (\hat{\omega} \times \vec{r})
\]
\[
\Rightarrow \vec{\omega} = \frac{\dot{r}}{r^2} \vec{w}_0, \quad \vec{\omega} = \hat{\omega}_0.
\]

3.32. For this problem, let's choose \(B\) axis as the axis about which the sphere is rotating. So, a point in the sphere \((r, \theta, \phi)\) has a distance to \(B\):
\[
d = r \sin \theta.
\]

We have the expression of the moment of inertia:
\[
I = \int r^2 \sin \theta \, dV = \frac{M}{V} \int r^4 \, dV \cdot \sin \theta \, d\phi \, ds \, d\theta = \frac{M}{\frac{4}{3} \pi} \cdot \frac{R^5}{2} \cdot \frac{r^4}{4} \cdot \sin \theta = \frac{2}{5} M R^2
\]

Note: Compared with 3.21, we could directly use spherical polar coordinates, that's because that the quantity calculated here has nothing to do with well-defined direction, or it's a quantity only showing the quality of distance.

3.34. Take the CM frame, in this frame, there is no torque about the CM point. So we conclude
that the angular velocity of the rod rotating around its CM point is a constant. So, to catch to rod when it rotates exactly $n$ rounds, we have:

$$t = n \cdot \frac{2\pi}{\omega_0} = \frac{2\pi V_0}{g} \implies V_0 = \frac{n \pi g}{\omega_0}$$

37 (a)

(b) \[ \sum m_w \cdot \vec{r} = \sum m_x \cdot \vec{r} - (\sum m_w) \cdot \vec{r} = (\sum m_w) \cdot \vec{r} = (\sum m_w) \cdot \vec{r} - (\sum m_w) \cdot \vec{r} = 0 \]

$\sum m_w \vec{r}$ is the position of the CM point in the frame which origin is just the CM point. So obviously $\sum m_w \vec{r} = 0$

(c) \[ \vec{L} \text{ (about CM)} = \sum m_w \vec{r} \times \dot{\vec{r}} \]

\[ \implies \vec{L} \text{ (about CM)} = \sum m_x \vec{r} \times \dot{\vec{r}} + \sum m_y \vec{r} \times \dot{\vec{r}} = \sum m \vec{r} \times \dot{\vec{r}} - (\sum m_w) \cdot \vec{r} \]

\[ = \sum m \vec{r} \times \dot{\vec{r}} = \sum m \vec{r} \times \vec{F}_{ext} + \sum \vec{r} \times \vec{F}_{int} \]

\[ \sum \vec{r} \times \vec{F}_{int} = \frac{1}{2} \sum \vec{F}_{int} \cdot \left( \vec{r} \times \vec{F}_{int} + \vec{r} \times \vec{F}_{int} \right) = \frac{1}{2} \sum \vec{F}_{int} \cdot \left( \vec{r} \times \vec{F}_{int} \right) \]

For central internal forces, we have \[ \sum \vec{r} \times \vec{F}_{int} = 0 \]

\[ \implies \vec{L} \text{ (about CM)} = \sum m \vec{r} \times \vec{F}_{ext} = \int \vec{p} \text{ (about the CM)} \]