Problem (1.)

An asteroid is observed to be orbiting around a gaseous cloud in such a way that its angular momentum $L$ and kinetic energy $K$ satisfy the equation $K/L^2 = A_0 \exp[2\theta]$. Use circular coordinates in the following question.

(a.) Using the inverse of the radial distance, (i.e. $r = u^{-1}$) and $\theta$ as the independent variable, derive an expression for $K/L^2$ and use it to find $u(\theta)$.

(b.) If the angular momentum is given by $L_0 \exp[-2(t/\tau_0)]$ find the potential energy for this problem.

Problem (2.)

Consider two pieces of wire each with mass $M_w$ (with uniform densities). Both are bent into the shape of a semi-circle. One wire ‘sits’ in the $x$-$y$ plane and the other in the $x$-$z$ plane. Both wires have their ends resting on the points (-4, 0) and (4, 0).

(a.) What is the location of the center of mass?

(b.) What is the moment of inertia tensor for the system of wire?

(c.) What is the rotational kinetic energy of the wire system?
Problem (3.)

A Klingon Warbird and the Starship Enterprise are approaching the planet Vulcan but from opposite directions. Each has standard ‘running lights’ (flashing lights on the exterior). Each captain sends a message to a scientist on the planet that the frequency of their light is 430 trillion Hz. However, using a powerful sensors the scientist observes the following data

<table>
<thead>
<tr>
<th></th>
<th>Mass</th>
<th>Frequency</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enterprise</td>
<td>190 million kg</td>
<td>680 trillion Hz</td>
<td>1000 m</td>
</tr>
<tr>
<td>Warbird</td>
<td>200 million kg</td>
<td>720 trillion Hz</td>
<td>1250 m</td>
</tr>
</tbody>
</table>

(a.) From the deck of the Enterprise, how fast does the Warbird appear to be approaching?

(b.) From the deck of the Warbird, what would be the observed mass of the Enterprise?

Problem (4.)

Imagine you are in an airplane that is slowly spiraling in for a landing. The position vector of the airplane as described by someone on the ground is given by

\[ \vec{R}_p = \rho_0 \cos(\omega_0 t) \hat{x} - \rho_0 \sin(\omega_0 t) \hat{y} + \left[ H_0 - v_0 t \right] \hat{z} \]

(a.) What are the acceleration and velocity vectors of the airplane?

(b.) When the airplane lands, how many complete rotations and fractions thereof has it made?

(c.) Now sitting in your seat facing the front of the plane, you decide to define mutually orthogonal unit vectors with you at their origin. The direction when you look straight ahead is your ‘x-direction’ and the direction of your left shoulder is your ‘y-direction.’ Finally your ‘z-direction’ is perpendicular to these two. Write explicit expressions for ‘your’ unit vectors as described by someone on the ground.

(d.) In your frame of reference, write Newton’s Second Law.
Problem (5.)

A bead of mass $M$ is constrained to slide along the frictionless surface of a sphere of radius $R_0$. There is a potential energy associated with the position of the given by

$$ U(\vec{r}) = M A_0 [\ell_1 x + \ell_2 y + \ell_3 z] $$

(a.) Find the Lagrangian for this system and express it in terms of two angles and their time derivatives.

(b.) Find the equation of motion of this system via the Euler-Lagrange equations.

(c.) Find the Hamiltonian of this system.

Problem (6.)

Now two particles of mass $M_1$ and $M_2$ with coordinates $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ are constrained to the surface of the same sphere. The potential energy of the new system is given by

$$ U_{Total} = U(\vec{r}_1) + U(\vec{r}_2) + \frac{1}{2} k_A R_0^2 (6\theta_1 - 5\theta_2)^2 + \frac{1}{2} k_B R_0^2 (\theta_1)^2 + \frac{1}{2} k_C R_0^2 (\theta_2)^2 $$

(a.) What is the form of Newton’s second law?

(b.) Solve completely to describe the equation of motion for this system. Include any appropriate discussion of normal modes, eigenmodes and eigenfrequencies.