Chapter 4 assignment: Read chapter 4, then do these problems in chapter 4:

1. K+K, Chapter 4, Problem 1
2. K+K, Chapter 4, Problem 2
3. K+K, Chapter 4, Problem 5
4. K+K, Chapter 4, Problem 7

Problems in classical statistical mechanics:
5. A pendulum has length L and mass m, and makes an angle $\theta$ with the vertical direction. Assuming that the amplitude of oscillation is small, find $<\theta>$, $<\theta^2>$, $<\nu>$, and $<\nu^2>$ when the temperature of the pendulum is $\tau$. Hint: Express the Hamiltonian in terms of the single coordinate ($\theta$) and conjugate momentum (angular momentum $\ell$) of the pendulum. Expand the potential for small oscillations, but take the limits of integration on $\theta$ in the partition function out to $\pm\infty$.

6. Referring to problem 5, what value of mL will give $<\theta^2>^{1/2} = 0.001$ radian when the pendulum is at $T = 300$ K?

7. Consider a classical N-particle system with Hamiltonian given by

$$H = \sum_{i=1}^{N} \left( \frac{p_{xi}^2 + p_{yi}^2 + p_{zi}^2}{2m_i} \right) + V(r_1, r_2, \ldots, r_N)$$

where $V$ is the potential energy in terms of the coordinates of all the particles. Show that $Z$ can be separated into kinetic and potential energy parts, $Z = Z_{\text{kin}}Z_{\text{pot}}$, and that the $Z_{\text{kin}}$ term is the partition function for an ideal gas. This result is useful for studying the statistical mechanics of liquids, because $Z_{\text{pot}}$ depends only on the positions of the particles.

General hints: Problems 5-7 involve classical statistical mechanics. In this case, the sums over quantum states are replaced by integrals over all position and momentum components, so that, for example,

$$\langle X \rangle = \frac{1}{h^{N_{\text{d}}}Z} \int \cdots \int X(\vec{p}, \vec{q}) e^{\left( \frac{-H(\vec{p}, \vec{q})}{\hbar} \right)/\tau} (d\vec{p})^N (d\vec{q})^N$$

$$Z = \frac{1}{h^{N_{\text{d}}}Z} \int \cdots \int e^{\left( \frac{-H(\vec{p}, \vec{q})}{\hbar} \right)/\tau} (d\vec{p})^N (d\vec{q})^N$$
Here $N$ is the number of particles, and $d$ is the spatial dimensionality, of the problem. Note that there is one integral for each component of momentum and position for each particle, so that there are $2N\times d$ integral signs indicated by the $\int \ldots \int$ in the formulas.