1) In class and in Griffiths the problem of hard-sphere scattering was considered. It was found that the partial wave coefficients $c_i = -i\sqrt{4\pi (2l + 1)} \frac{j_i(ka)}{h_i^{(1)}(ka)}$ where $a$ is the radius of the sphere.

a) Using the explicit form for $j_0(ka)$ and $h_0^{(1)}(ka)$ show that $c_0 = i\sqrt{4\pi} \sin(ka)e^{-ika}$.

b) Show that this means that the phase-shift is given by $\delta_0 = -ka$. The fact that we can define a (real) phase shift implies means the value of $c_0$ has an absolute upper bound. This bound followed from unitarity (conservation of particle number). Thus our solution is consistent with unitarity.

c) At very low incident momentum ($ka << 1$) the $l=0$ partial wave dominates the scattering. Show that as $ka$ goes to zero the differential cross-section is given by $\frac{d\sigma}{d\Omega} = a^2$, independent of angle.

2) Use the result of part 1.c to show that if a total of $g$ particles per unit area are incident on the target (where $g = \int d\Omega \text{flux}(t)$) with low incident momentum, then the total number of particles scattered in the backward hemisphere ($\theta > \pi/2$) is given by $g 2\pi a^2$.

Problem 11.4