1. In class we developed time dependent perturbation theory using the interaction representation. The purpose of this problem is to show that the transition probability for going from state $a$ to $b$ which is given by $P_{a\to b} = \langle a | U(t) | b \rangle$ in this formalism is identical to that found by the formulae in the book. In particular consider the case studied in the book: a two level system with $H(t) = H' = 0$. For this case show that $P_{a\to b}$ and $P_{a\to a}$ (the probability that the system starts in $a$ and remains in $a$) as calculated from the interaction representation at both first order and second order in perturbation theory agrees with the expressions in the book.

2. For the case studied in the book (a two level system with $H(t) = H' = 0$ starting with system in level $a$) show explicitly that unitarity is maintained to the order at which we work. By maintaining unitarity at a given order, I mean check to make sure that the total probability of being in a state adds up to one plus an error which is smaller than order to which we work. To keep track of orders replace $H'$ by $\lambda H'$ and keep track of powers of $\lambda$. Verify this at both first and second order. At first order this is very easy; at second order this is not so trivial to show. The easiest way is to look at the quantity $\frac{d}{dt} \left( |c_a|^2 + |c_b|^2 \right)$ and show that it vanishes at second order. This in turn says that $|c_a|^2 + |c_b|^2$ does not change with time at this order and hence equals its value at $t=0$ which is unity.

Griffiths 9.6, 9.