Homework 1 Due Friday, February 6, 2009 @ 9 AM

1. Griffiths, 2nd Edition, Problem 4.1 (a) only Commutation relations \([r_i, p_j]\), etc.

2. Griffiths, 2nd Edition, Problem 4.2 (a) and (b) only Separation of variables in a 3D Cartesian infinite cubical well. Find the eigenfunctions and eigenvalues. Determine the degeneracies of some of the lowest-energy states.


4. Griffiths, 2nd Edition, Problem 4.13 (a) and (b) only Use the H-atom GS WF to calculate expectation values \(<r>, <r^2>, <x>, <x^2>\)

5. Griffiths, 2nd Edition, Problem 4.19 (a) and (b) only Ang. Mom. commutation relations \([L_z, x], [L_z, p_z], [L_z, L_x]\), etc.

6. Griffiths, 2nd Edition, Problem 4.22 (a) and (b) only Ang. Mom. raising operator \(L_+\) and \(Y_{\ell}^m\). THIS PROBLEM IS NOW DUE WITH HW#2

Extra Credit 1 Schrod. Eq. in 3D → Separate variables → \(\theta\)-equation → change of variables \(x=\cos\theta\), etc. → Associated Legendre equation → take \(m=0\) and use series solution method around \(x=0\) → keep solution finite at \(x=±1\) → find \(\ell\) must be an integer

Extra Credit 2 Radial equation → substitute \(u(r) = rR(r)\) → find asymptotic behavior of \(u(r)\) → find new equation for \(v(r)\) → solve by series solution → find condition to keep solution normalizable → find eigen-energies of H-atom

Office Hours Thursday, 3:00 – 4:30 PM, Room 0360
(see class web site for directions to the room)

TA (Wai-Lim Ku) Office Hours, Thursday 4:30 – 5:30 PM, Room 0104
1. Consider the solutions to the radial part of the Schrödinger equation for the hydrogen atom, \( R_n^\ell(r) \). Note that the radial part of the probability density is proportional to \( |r R_n^\ell(r)|^2 \).

**a)** Figure out a general expression for the number of zeros in \( R_n^\ell(r) \), excluding those at \( r = 0 \) and \( r = \infty \), in terms of \( n \) and \( \ell \).

**b)** Sketch the effective potential for \( \ell = 0 \) and \( \ell = 1 \) and draw several bound states. Sketch solutions to the radial equation (given below) in terms of the “probability amplitude” \( r R_n^\ell(r) \) for \( \{n = 1, \ell = 0\} \), \( \{n = 2, \ell = 1\} \), and \( \{n = 3, \ell = 1\} \). The effective potential is the term in square brackets:

\[
\frac{-\hbar^2}{2m} \frac{d^2}{dr^2} (rR) + \left[ -\frac{\hbar^2}{4\pi\varepsilon_0} \frac{1}{r} + \frac{\ell(\ell + 1)}{2mr^2} \right] (rR) = E(rR)
\]

Use your knowledge of the asymptotic behavior of the solutions, as well as properties of solutions to one-dimensional differential equations, to make your sketches semi-quantitative.