1. In class we asserted that in quantum mechanics physical properties with Hermitian operators. These were defined an operator $\hat{A}$ as Hermitian operator if $\int dx \psi^* (\hat{A} \psi)$ is real for all square integrable wave functions $\psi$. In this problem, you will use this definition to derive the following properties of Hermitian operators:

   a. If $\int dx \psi_1^* (\hat{A} \psi_2) = \int dx \ (\hat{A} \psi_1)^* \psi_2$ for all square integrable wave functions $\psi_1, \psi_2$ then $\hat{A}$ is Hermitian.

   b. If $\hat{A}$ is Hermitian, then $\int dx \psi_1^* (\hat{A} \psi_2) = \int dx \ (\hat{A} \psi_1)^* \psi_2$ for all square integrable wave functions $\psi_1, \psi_2$. Hint: consider $\psi = \psi_1 + \psi_2$ and use the definition.

   Note that since the property $\int dx \psi_1^* (\hat{A} \psi_2) = \int dx \ (\hat{A} \psi_1)^* \psi_2$ is both necessary and sufficient for $\hat{A}$ to be Hermitian (as shown in a. and b.) this property is an alternative definition for Hermiticity.

   c. If $\hat{A}$ is Hermitian then $\hat{A}^2$ is also. Hint: consider $\psi = \hat{A} \phi$ and start with $\int dx \psi^* \psi$. which we know to be real and use the property in a).

   d. If $\hat{A}$ and $\hat{B}$ are Hermitian, then $\int dx \psi_1^* (\hat{A} \hat{B} \psi_2) = \int dx \ (\hat{B} \hat{A} \psi_1)^* \psi_2$ Hint use the property in part b. twice.

   e. If $\hat{A}$ and $\hat{B}$ are Hermitian, then $\hat{C} = \hat{A} \hat{B} + \hat{B} \hat{A}$ is Hermitian. Hint: use part d.

   f. If $\hat{A}$ and $\hat{B}$ are Hermitian, then $\hat{C} = i[\hat{A}, \hat{B}]$ is Hermitian. Hint: use part d.

2. Use the general form of the uncertainty relation to show that $\sigma_x \sigma_p \geq \hbar |\langle x \rangle|^2$ for any wavefunction. That is show that $\left( \langle x^4 \rangle - \langle x^2 \rangle^2 \right) \left( \langle p^2 \rangle - \langle p \rangle^2 \right) \geq \hbar |\langle x \rangle|^2$.

3. Show that in quantum mechanics $\frac{d\langle O \rangle}{dt} = i \frac{[\hat{H}, \hat{O}]}{\hbar}$ where $\hat{O}$ is a Hermitian operator associated with a physical observable that contains no explicit time dependence and $\hat{H}$ is the Hamiltonian operator. Hint: Use the Schrödinger equation in the form $\hat{H} \psi = i \hbar \frac{d\psi}{dt}$.