A. Exercise your math muscles

1) Compute $i^i$
2) Compute $e^{i\pi/2}$
3) Find the general solution to
\[ -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x), \]  
for $E > 0$.
4) What is the solution to the problem above satisfying the conditions
\[ \psi(0) = 1, \quad \frac{d\psi(x)}{dx}\bigg|_{x=0} = 0 ? \]

B. A first look at the Uncertainty Principle

Consider a particle described at some particular instant of time by the wave function $\psi(x) = Ae^{-ax^2}$.
1) Determine $A$ so $\psi$ is normalized.
2) Compute $\langle x \rangle$, $\langle x^2 \rangle$ and $\sigma_x^2 = \langle (x - \langle x \rangle)^2 \rangle$.
3) Compute $\langle p \rangle$, $\langle p^2 \rangle$ and $\sigma_p^2 = \langle (p - \langle p \rangle)^2 \rangle$.
4) Show that by changing $a$ one can make either $\sigma_x^2$ or $\sigma_p^2$ small, but not both at the same time. Compute $\sigma_x\sigma_p$.

C. Ehrenfest’s theorem

Prove that
\[ \frac{\partial}{\partial t} \langle p \rangle = \int_{-\infty}^{\infty} dx \, \psi^*(x,t) \left( -\frac{\partial V(x)}{\partial x} \right) \psi(x,t). \]

This result is one way to show that, under certain circumstances, macroscopic objects obey Newton’s law $F = ma$. Describe in words the connection of the formula above with Newton’s law.