Two finite QWs, decoupled by wide barrier.

Double well, each 0.44 nm wide, 1.3 nm apart.

Potential energy (eV) vs distance (nm):
- Potential energy: $V(x)$
- Distance: $x$
- Wide barrier indicated.

Electron density distributions:
- 1st and 2nd energy levels.
- Evanescent decay into classically forbidden region.
- No overlap.
- Identical probability density.

Wavefunctions:
- $\Psi_L$: electron in left well.
- $\Psi_R$: electron in right well.
- Symmetric $\propto \Psi_L + \Psi_R$.
- Antisymmetric $\propto \Psi_L - \Psi_R$.

Energy vs quantum number $n$:
- Two degenerate bound states with equal energy.
Two finite QWs, coupled by thin barrier.

Double well, each 0.44 nm wide, 0.2 nm apart.

New, barrier width comparable to the evanescent decay length.

Double well electron density distributions:
- 1st
- 2nd

Very different probability density.

Double well energy eigenvalues:
- "broken" degeneracy: different energy eigenvalues!

Antisymmetric must go to zero!

Double well: first two wavefunctions:
- 1st
- 2nd

Significant wavefunction overlap.
Two finite QWs: bound state "repulsion"

- Energy increased because $\Psi$ must go to zero → shorter wavelength $\lambda$ → higher $E$.
- Energy decreased because overlap in barrier increases long-wavelength components of $\Psi$ → smaller $\lambda$ and lower $E$.

Figure 1:
- "antibonding"
- "bonding"
- "broken degeneracy"
- "degenerate"
Exception to B.C. "\( \psi \) continuous"

Example: \( V(x) = -\alpha \delta(x) \) \( (\int \delta(x) \, dx = 1) \)

\[
- \frac{\hbar^2}{2m} \psi'' - \alpha \delta(x) \psi = E \psi
\]

\[
- \frac{\hbar^2}{2m} \int_{-\epsilon}^{\epsilon} \psi'' \, dx - \alpha \int_{-\epsilon}^{\epsilon} \delta(x) \psi \, dx = \int_{-\epsilon}^{\epsilon} E \psi \, dx
\]

\[
\lim_{\epsilon \to 0} \left[ \frac{\hbar^2}{2m} \left( \psi' \big|_{x=\epsilon} - \psi' \big|_{x=-\epsilon} \right) - \alpha \psi(0) \right] = 0
\]

\[
\left| \psi' \right|_{x=\epsilon} - \left| \psi' \right|_{x=-\epsilon} = -\frac{2mE}{\hbar^2} \psi(0) \quad \text{NOT zero!}
\]

Note: Since \( \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \), since \( dx \) has units of length, \( \delta(x) \) must have units of length\(^{-1}\). Therefore, since \( V(x) \) has units of energy, \( \alpha \) has units of Energy \cdot length.