Physics 401 - Homework #1 - Due Wednesday September 9th

1) (4 points total) Convert the following complex numbers into the $z = x + iy$ and $z = Ae^{i\theta}$ forms. State explicitly the values for $x$, $y$, $A$, and $\theta$ for each.

   a) $i^i$
   b) $\frac{1}{1-i}$

2) (4 points total) Prove the following identities for complex numbers:
   a) $\text{Re}(z) = (z + z^*)/2$
   b) $\text{Im}(z) = (z - z^*)/2i$
   c) $\cos(\theta) = \frac{\exp(i\theta) + \exp(-i\theta)}{2}$
   d) $\sin(\theta) = \frac{\exp(i\theta) - \exp(-i\theta)}{2i}$

3) (3 points total)
   a) Write down the phase of the following wavefunction:
      $$\Psi(x,t) = Ae^{i(kx-\omega t)}$$
   b) Show that the phase is shifted by $\pi/2$ when the function is multiplied by $i$
   c) Show that the phase is shifted by $\pi$ when multiplied by $-1$.

4) (3 pts) Complex numbers are useful for quickly deriving trigonometric identities. Prove the laws of addition of sines and cosines
   $$\sin(\theta + \varphi) = \sin \theta \cos \varphi + \cos \theta \sin \varphi$$
   $$\cos(\theta + \varphi) = \cos \theta \cos \varphi - \sin \theta \sin \varphi$$
   using Euler’s formula ($\exp(i\theta) = \cos \theta + i \sin \theta$) and exponential notation. Hint: work out $\exp(i(\theta + \varphi))$, and then take the real and imaginary parts.

5) Classical harmonic oscillator. (5 points total) A particle of mass ($m$) is attached to a spring. The spring exerts a force on the mass proportional to the displacement from equilibrium, and the force acts opposite to the displacement (it is a restoring force). Therefore the force is described by $F = -kx$, where ($k$) is the spring constant, and ($x$) is the displacement measured from the equilibrium position.
a) Write down Newton's second law of motion for this system as a differential equation. The equation should be satisfied by a function x(t) which describes the particle's position as a function of time.

b) Find a general solution to the equation.

c) What is the angular frequency of the oscillator in terms of (k) and (x)?

d) How many initial conditions are necessary to specify a particular solution?

e) Choose a set of initial conditions and find the corresponding particular solution. Please don't choose the trivial solution: x(t) = 0!. 