Experiment 8

The Michelson Interferometer

1 Introduction

There are, in general, a number of types of optical instruments that produce optical interference. These instruments are grouped under the generic name of interferometers. In this and the next laboratory, you will study two different interferometers: the Michelson interferometer and the Fabry-Perot interferometer. The Michelson interferometer causes interference by splitting a beam of light into two parts. Each part is made to travel a different path and brought back together where they interfere according to their path length difference.

You will use the Michelson interferometer to observe the interference of two light sources: HeNe laser and sodium lamp. You will study interference patterns quantitatively to determine the wavelengths and splitting of the Na D lines empirically. You will use the HeNe laser interference spectrum to calibrate the interferometer.

2 Theory

In this section, you will learn about the theory of interference of light waves. Conditions for the occurrence of constructive and destructive interference will be derived. Next, the theoretical operation of the Michelson interferometer will be discussed. You will learn how the device produces interference patterns.

2.1 Interference of Waves With a Single Frequency

Light waves consist of electric and magnetic fields. As an electromagnetic wave propagates, both the magnitude and direction of its electric and magnetic fields vary in space and time. The electric and magnetic fields lie in a plane perpendicular to the direction of propagation of the
wave and are perpendicular to each other. An example of a light wave propagating in the x-direction and the oscillation of its electric field in the y-direction is shown in Fig. 8.1.

If two waves should simultaneously propagate through the same region of space, the resultant electric field at any point in that region is the vector sum of the electric field of each wave. This is an illustration of the principle of superposition. An example is shown in Fig. 8.2. Here, two waves of different wavelengths, shown by dotted lines, interfere. The resultant wave is the vector sum of the two waves, and is indicated by the solid line.

As we will see below, if the phase difference between the two waves is an integer multiple of $2\pi$, the disturbances are said to be in phase and total constructive interference occurs. Here, the intensity of the resultant electric field is a maximum. If the phase difference between the two waves is an odd multiple of $\pi$, total destructive interference occurs. Here, the intensity of the resultant electric field is zero.

To understand interference a bit more concretely, consider two electromagnetic waves of the same wavelength, $\lambda$, or frequency, $\omega$, originating at point $X$ in Fig. 8.3. One wave travels to point $Y$ along path 1 and the other travels to point $Y$ along path 2. The length of path 1 is $r_1$ and the length of path 2 is $r_2$. Furthermore, let the two waves originating at $X$ have the same amplitude and phase. The electric field of these waves at point $Y$, at a particular instant in time, can be written as

\[
\begin{align*}
E_1(Y) &= E_0 \cos \left( \psi_X + \frac{2\pi}{\lambda} r_1 \right), \\
E_2(Y) &= E_0 \cos \left( \psi_X + \frac{2\pi}{\lambda} r_2 \right).
\end{align*}
\]

The difference in optical length between the two paths is known as the optical path difference, which we will denote by $\Delta r = r_2 - r_1$. A related quantity is the phase difference, $\Delta \phi$, given by

\[
\Delta \phi = \frac{2\pi}{\lambda} \Delta r.
\]  

(1)

The total electric field at point $Y$ is given by

\[
\begin{align*}
E_T(Y) &= E_1(Y) + E_2(Y) \\
&= E_0 \left[ \cos \left( \psi_X + \frac{2\pi}{\lambda} r_1 \right) + \cos \left( \psi_X + \frac{2\pi}{\lambda} (\Delta r + r_1) \right) \right].
\end{align*}
\]  

(2)

For constructive interference at point $Y$, the amplitude of $E_T(Y)$ must be a maximum. This occurs when

\[
\cos \left( \psi_X + \frac{2\pi}{\lambda} r_1 \right) + \cos \left( \psi_X + \frac{2\pi}{\lambda} (\Delta r + r_1) \right) = \pm 2.
\]
This condition occurs when

$$\Delta \phi = 2m\pi, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots.$$ (3)

For destructive interference at point \( Y \), the amplitude of \( E_T(Y) \) must be zero. This occurs when

$$\cos \left( \psi_X + \frac{2\pi}{\lambda} r_1 \right) + \cos \left( \psi_X + \frac{2\pi}{\lambda} (\Delta r + r_1) \right) = 0.$$ 

This condition is achieved when

$$\Delta \phi = \pm (2m + 1) \pi, \quad m = 0, 1, 2, 3, \ldots.$$ (4)

### 2.2 Interference of Waves with Two Frequencies

We will now consider the case of two frequencies, \( \omega_1 \) and \( \omega_2 \), or two wavelengths, \( \lambda_1 \) and \( \lambda_2 \). First, consider a beam propagating along the \( x \) axis before it enters the interferometer. It turns out to be easier to consider the complex notation for this discussion. It should be remembered, however, the physical fields are obtained by taking the real parts of the following expressions. The result of the sum of two complex waves with different amplitudes at point \( x \) is given by:

$$E_T = E_1 e^{ik_1 x} + E_2 e^{ik_2 x}$$

$$= E_1 \left( e^{ik_1 x} + \frac{E_2}{E_1} e^{ik_2 x} \right),$$ (6)

where we have used the shorthand of \( k \) for \( 2\pi/\lambda \). If we let \( a = E_2/E_1 \), we can rewrite \( E_T \) as

$$E_T = E_1 e^{i\frac{k_1+k_2}{2}x} \left\{ (1-a) e^{i\frac{k_1-k_2}{2}x} + a \left[ e^{i\frac{k_1-k_2}{2}x} + e^{-i\frac{k_1-k_2}{2}x} \right] \right\}$$

$$= E_1 e^{i\frac{k_1+k_2}{2}x} \left\{ (1-a) e^{i\frac{k_1-k_2}{2}x} + 2a \cos \frac{k_1 - k_2}{2} x \right\}.$$ (8)

This describes a field with a fast oscillation with spatial frequency \((k_1 + k_2)/2\) and a much slower envelope with frequency \((k_1 - k_2)/2\). Here, we are interested in the intensity, \( I_T \), of the so-called beat envelope, which can be obtained by multiplying \( E_T \) by its complex conjugate. After some manipulation and assuming \( E_1 \) and \( E_2 \) are real, the intensity takes the form

$$I_T = E_1^2 (1 + a) \left[ 1 - a \cos \left( \frac{k_1 - k_2}{2} x \right) \right].$$ (9)

This intensity that varies as we move along \( x \) as \( \cos((k_1 - k_2) x/2) \), with beat frequency \((k_1 - k_2)/2\). The intensity is maximum when

$$(k_1 - k_2) x = \left( \frac{2\pi}{\lambda_1} - \frac{2\pi}{\lambda_2} \right) x = (2m + 1) \pi$$

$$= (2m + 1) x.$$ (10)
and minimum when
\[
\left( \frac{2\pi}{\lambda_1} - \frac{2\pi}{\lambda_2} \right) x = 2m\pi
\]
where \( m = 0, 1, 2, \ldots \). When such a wave is sent through the interferometer we again see interference as described in the previous section but with a variation given by \( \cos(k_1 + k_2) \Delta x \). Now however, beat envelope causes the contrast of the fringe pattern to vary periodically as well.

### 2.3 The Michelson Interferometer

The Michelson interferometer is a device that produces interference between two beams of light. A diagram of the apparatus is shown in Fig. 8.4. The basic operation of the interferometer is as follows. Light from a light source is split into two parts. One part of the light travels a different path length than the other. After traversing these different path lengths, the two parts of the light are brought together to interfere with each other. The interference pattern can be seen on a screen.

Light from the source strikes the beam splitter (designated by \( S \)). The beam splitter allows 50\% of the radiation to be transmitted to the translatable mirror \( M_1 \). The other 50\% of the radiation is reflected to the fixed mirror \( M_2 \). The compensator plate \( C \) is introduced along this path to make each path have the same optical path length when \( M_1 \) and \( M_2 \) are the same distance from the beam splitter. After returning from \( M_1 \), 50\% of the light is reflected toward the frosted glass screen. Likewise, 50\% of the light returning from \( M_2 \) is transmitted to the glass screen. At the screen, the two beams are superposed and one can observe the interference between them.

### 3 Experiment

In the following experiments, you will use the Michelson interferometer to observe interference of light. The sources of light you will use are a HeNe laser and a sodium lamp. You will be able to determine the wavelength of the light from each source by studying the interference patterns quantitatively.

#### 3.1 HeNe Laser Light

In this part of the experiment, you will use the HeNe laser as your light source. Inject the laser beam into the Michelson interferometer. Make sure the beam is properly retro-reflected. Initially, you will see two bright spots on the screen. Adjust the angle of the fixed mirror until
these two spots overlap.

Now, insert two plano-concave lenses between the laser and the interferometer as shown in Fig. 8.5; they should be approximately 15 cm apart. These lenses are used to expand the laser beam, making the interference pattern more visible. The fixed mirror, $M_2$, should be adjusted until you can see concentric circles of light and dark fringes (i.e., a bulls-eye pattern) on the frosted glass screen. **Note, take care when moving this $M_2$ as the interference is very sensitive to its alignment.** As you translate mirror $M_1$, you will see fringes appearing and disappearing on the screen. The apparatus is set up in such a way that the wavelength of the light can be found using the equation

$$
\lambda = \frac{1}{5} \left( \frac{2d}{m} \right) 
$$

(12)

where $d$ is the distance mirror $M_1$ was moved and $m$ is the number of rings that disappeared (or appeared) while $M_1$ was being moved.

Although it is possible to count the rings to estimate the wavelength, we will estimate it by acquiring data with the computer. Proceed in the following way. Once you have produced a bulls-eye interference pattern using the beam expanders, remove them from the apparatus. When you do this, you will see only a small spot of red light. Let this spot shine into the center of the photodiode. Use the synchronous motor to facilitate the turning of the micrometer. As the micrometer is turning, record the interference data with the computer. From this data, you can deduce $d$ from the fourier transform of the spectrum, and a nonlinear fit of the fringe pattern. The following approximate data will allow you to estimate the speed of the mirror and thus allow you to estimate $d$:

a. The speed of the synchronous motor is 0.5 rpm.

b. The interferometer lever arm reduction factor is 5X.

c. The micrometer screw moves $5 \times 10^{-4}$ m/rev.

Since this data is only approximate, use the known wavelength of your HeNe laser to calibrate the speed more accurately.

### 3.2 Sodium Light

Now use the sodium lamp to produce an interference pattern. Since the spectrum of this light consists primarily of two closely spaced lines (a doublet), each wavelength will produce its own set of fringes. The wavelengths of the doublet will be labeled by $\lambda_1$ and $\lambda_2$. The separation
between the wavelengths will be denoted by $\Delta \lambda = \lambda_1 - \lambda_2$; it will also be helpful to introduce the wavelength $\lambda$, given as

$$\lambda = \sqrt[3]{\lambda_1 \lambda_2}$$

(13)

Your goal will be to empirically determine $\lambda_1$ and $\lambda_2$ by measuring $\lambda$ and $\Delta \lambda$. You can determine $\lambda_1$ and $\lambda_2$ by using these measured values and Eq. (13).

The experimental setup for this part of the laboratory is shown in Fig. 8.6. Once you have assembled the apparatus, you should manually move mirror $M_1$ and notice the following: as you move $M_1$, you will, at times, notice that the interference pattern consists of concentric circles of highly contrasted bright and dark fringes. This occurs because the bright fringes produced by one wavelength are falling on the bright fringes produced by the other wavelength. The same goes for the dark fringes. As you continue to move $M_1$, you will notice that the interference pattern becomes uniformly illuminated. This occurs because the bright fringes of one wavelength are overlapping the dark fringes of the other wavelength, and vice versa. These patterns repeat themselves as you continue to move $M_1$. This type of pattern is indicative of the beats we discussed earlier.

To measure $\lambda$, you should project the interference pattern onto the photodiode and attach the motor to the micrometer screw. Use the computer to take data. Be sure to take enough data to observe at least one entire cycle of the beat envelope. Taking 30,000 data points at 30 Hz will be suitable. Note how long it will take to acquire this data. You should shield your detector appropriately so that your spectrum will not be adversely affected if others in the room turn on and off their desk lamps and/or leave the room. The Fourier transform of your data will give you an approximation to $\lambda$. Be sure to use the speed you calibrated in the previous section for this determination.

To measure $\Delta \lambda$ from your spectrum, estimate the distance the mirror in one arm had to move for the beat envelope to repeat. You can determine $\lambda_1$ and $\lambda_2$ by solving the expressions for $\lambda$ and $\Delta \lambda$ simultaneously.

3.2.1 Sodium Light: Extra Credit

Determine the distance through which mirror $M_1$ must be moved to significantly diminish the contrast of the fringe pattern even at locations of maximum contrast due to $\lambda_1$ and $\lambda_2$. The expected distance is on the order of 0.5 cm - 1.0 cm for such a high pressure sodium lamp, and is known in optics as the longitudinal coherence length of the source.

4 Computer Acquisition of Data
This section will discuss how a computer will be used to sample the voltage output of a photo-detector on which light from the Michelson interferometer is incident.

4.1 Connections
The connections between the experimental apparatus and the computer are made in the following way:

1. Connect the signal output from the photodiode power supply to the signal input on the amplifier-filter module.
2. Connect the signal output from the amplifier-filter module to the signal input (channel 0) on the computer interface module.
3. Connect the photodiode ground terminal (on the photodiode head) to the ground terminal on the computer interface module.
4. Turn the power and filter of the amplifier-filter module on. Set the gain of that module to unity.
5. Move the motor stand so that the drive is mechanically disconnected from the micrometer screw of the interferometer. Turn on the computer.

4.2 Laser Light Source
The experimental procedure for analyzing the laser light is given below.

1. Move the motor stand so that the drive is mechanically connected to the micrometer screw of the interferometer. The height of the motor may need to be adjusted.
2. Set the motor switch (along the electrical cord) to the “on” position during data taking periods.
3. Start ACQUIRE.
4. For this experiment, it may be appropriate to start with a sample rate of 20-50 samples/sec and the total number of measurements of about 5,000.
5. The gain code can be set to the useful starting point of -5 to +5 volts. This can be changed as needed.

4.3 Sodium Light Source
The following describes how to use the computer to analyze the sodium light source.
1. Make sure the sodium light source is aligned with the interferometer as discussed in section 3.2.

2. Disengage the motor from the interferometer micrometer screw. Turn the micrometer screw by hand until the interference pattern consists of fringes of maximum contrast. Center this pattern on the face of the photodiode.

3. Re-engage the motor to the micrometer screw.

4. Set the gain on the amplifier-filter module to 100X and keep the filter active.

5. Set the gain code in the program ACQUIRE to the range -0.5 to +0.5 volt.

6. Make a test run with the sampling rate of 200 samples/sec and the total number of samples on the order of 3,000. Actual runs should be made between 20 and 50 Hz with a total number of samples of about 30,000.

For Further Reading:


