1. Pedrotti$^3$, 3rd edition, problem 2-7 (see Fig. 2-33).

**Solution:**

See FIGURE 2-33 in the text $P^3$

From the geometry it is clear that $\tan \theta_c = \frac{D/4}{h}$, where $h$ is the height of the slab and $D$ is the diameter of the circle of light. From Snell’s law we know that the critical angle occurs when the angle of refraction is $\theta_r = \frac{\pi}{2}$. Then applying Snell’s Law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$ we have:

$$n_{\text{glass}} = \frac{n_{\text{air}} \sin \pi/2}{\sin \theta_c} = \frac{D/4}{\sqrt{(D/4)^2 + h^2}} = 1.55$$

Where I used $n_{\text{air}} = 1$.

2. Write an expression for the $E$ - and $B$ -fields that constitute a plane harmonic wave traveling in the $+z$-direction. The wave is linearly polarized with its plane of vibration at $45^\circ$ to the $yz$-plane.

**Solution:**

For a plane wave traveling in the $+z$-direction we know the functional form of the wave must be $\sin(kz - \omega t)$ or cosine. Since the wave is traveling in free space, it must be transverse. This implies that $E_z = 0$. For light polarized linearly at a $45^\circ$ the normalized polarization vector is $\frac{1}{\sqrt{2}} (\hat{x} + \hat{y})$. Thus for a given amplitude $E_0$ we have for the equation of the electric field:

$$\vec{E}(z,t) = \frac{E_0}{\sqrt{2}}(\hat{x} + \hat{y})\sin(kz - \omega t)$$

Then from Ampere’s Law with no source term ($\vec{J} = 0$), $\vec{\nabla} \times \vec{B} = -\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$ it follows that $\hat{k} \times \vec{B} = \vec{E} / c$. From which the equation for the magnetic field follows:

$$\vec{B}(z,t) = \frac{E_0}{c\sqrt{2}}(\hat{y} - \hat{x})\sin(kz - \omega t)$$
3. Prove that to someone looking straight down into a swimming pool, any object in the water will appear to be \( \frac{3}{4} \) of its true depth.

**Solution:**

Consider the case where we are not looking directly down, but our line of sight is displaced a distance, \( x \). Then if the real object depth is \( d \) then the apparent object depth is \( a \). From the geometry in the picture we conclude that:

\[
\sin(\theta_i) = \frac{x}{\sqrt{x^2 + a^2}} \quad \text{and} \quad \cos(\pi/2 - \theta_i) = \sin(\theta_i) = \frac{x}{\sqrt{x^2 + a^2}}
\]

Then applying Snell’s law \( n_1 \sin(\theta_i) = n_2 \sin(\theta_2) \), we find:

\[
\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{\sqrt{x^2 + a^2}}{\sqrt{x^2 + d^2}}
\]

In the limit of looking straight down, we let \( x \to 0 \). And we find plugging in the values of the indices of diffraction: \( \frac{a}{d} = 1/1.333 = 0.75 \)

4. Light is incident in air perpendicularly on a sheet of crown glass having an index of refraction of 1.552. Determine both the reflectance and the transmittance.

**Solution:**

The equations for reflectance and transmittance at perpendicular incidence as gotten from Fresnel’s Equations are:

\[
R = \left( \frac{n_1 - n_t}{n_1 + n_t} \right)^2 \quad \text{and} \quad T = \frac{n_t}{n_1} \left( \frac{2n_t}{n_1 + n_t} \right)^2
\]

Plugging in the numbers we find: \( R=0.047 \) and \( T=0.953 \). Notice that \( R + T = 1 \), by energy conservation.

5. Show analytically that a beam entering a planar transparent plate, as in the figure, emerges parallel to its initial direction. Consider the case where the plate has a side length \( t \), and the laser beam has an angle of incidence \( \alpha \), and angle of refraction at the
first interface of $\beta$. Find an expression for the lateral displacement of the exiting beam relative to the incident beam, $s$, in terms of $t$ and trigonometric functions of $\alpha$ and $\beta$. Use Snell’s law and some geometrical thinking.

Solution:

From the picture we see that $\sin(\alpha - \beta) = s/ L$ and that $\cos \beta = t / L$. Thus:

\[
 s = \frac{t \sin(\alpha - \beta)}{\cos \beta} = \frac{t (\cos \beta \sin \alpha - \cos \alpha \sin \beta)}{\cos \beta} = t \sin \alpha \left( 1 - \frac{\tan \beta}{\tan \alpha} \right)
\]