1. A single slit in an opaque screen 0.10 mm wide is illuminated (in air) by plane waves from a krypton ion laser ($\lambda_0 = 461.9$ nm). If the observing screen is 1.0 m away, determine whether or not the resulting diffraction pattern will be of the far-field variety and then compute the angular width of the central maximum.

Solution:

Far field condition is given by $R > \frac{b^2}{\lambda}$, where $R$ is the distance from slit to observation point and $b$ is the slit width. $R=1.0$ m, and plugging in the numbers we find: $\frac{b^2}{\lambda} = 0.02 m$. YES, it is far field. From the diffraction formula we know the first zero of intensity is given by $\sin \theta = \frac{\lambda}{b}$. Implying: 

$$\theta = \sin^{-1} \left( \frac{\lambda}{b} \right) = 0.26^\circ$$

And the angular width is given by: $2\theta = 0.52^\circ$.

2. What is the relative irradiance of the subsidiary maxima in a three-slit Fraunhofer diffraction pattern? Draw a graph of the irradiance distribution, when the split spacing $a = 2b$, where $b$ is the slit width, for 2 and then 3 slits.

Solution:

The multi-slit diffraction formula is $I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin N\alpha}{\sin \alpha} \right)^2$. The subsidiary maxima occur for $N\alpha = p\pi/2$, with $p$ an odd integer and $\sin \alpha \neq 0$. Thus the first subsidiary maximum occurs of $p=3$ and we have $N=3$, thus $\alpha = \pi/2$. Using the fact that $\alpha/\beta = a/b$ and from the diffraction formula we find:

$$\frac{I(\theta)}{I_0} = \frac{1}{N^2} \left( \frac{\sin \beta}{\beta} \right)^2 \bigg|_{\alpha=\pi/2} \approx 1/9$$

Where the $1/N^2$ comes from the fact that the maximum $I$ is given by $N^2I_0$. 

![Graph of irradiance distribution for 2 and 3 slits.](image-url)
Above are plots of the diffraction pattern for a (left) 2-slit ($b = 0.04 \text{ mm}$, $a = .125 \text{ mm}$) and (right) 3-slit ($b = 0.04 \text{ mm}$, $a = .125 \text{ mm}$) illuminated with He-Ne laser light at 632.8 nm.

4. Pedrotti$^3$, 3rd edition, problem 11-3. See Fig. 11-19 on page 290.

Solution:
See figure 11-19.

a) The positions of the minima are given by: $m\lambda = b\sin \theta_m \approx b\theta_m = y_m / L$.

Where $L=2m$ and $y_m$ is the location of the $m$-th zero. Thus $\Delta y = y_3 - y_{-3} = 6\lambda L / b$.

Upon plugging in the numbers we find $m = 13.0$.

b) Again $L > \frac{b^2}{\lambda}$ is the far field condition and $\frac{L}{b^2 / \lambda} = 139$. YES this is far field.


Solution:
The full angular breadth of the central maximum is given by $\phi = 2\theta$, where $\theta$ is the position of the first zero. Thus,

$$b = \frac{\lambda}{\sin(\phi/2)}$$

For $\phi = 30^\circ, 45^\circ, 90^\circ, 180^\circ$ we have $b = 2.125 \text{ mm}, 1.437 \mu m, 0.778 \mu m, 0.55 \mu m$ respectively.


Solution:
The Airy disc formed by a circular diffraction aperture of diameter $D$ has angular radius: $\Delta \theta_{1/2} = \frac{1.22\lambda}{D}$. Thus the radius $R$ of the Airy disc formed is given by:

$$R = L \tan \Delta \theta_{1/2} \approx L \Delta \theta_{1/2} = 4.86 \times 10^6 \text{ m}$$

The irradiance is then given by:

$$I = \Phi / A = \frac{2000 W}{\pi (4.86 \times 10^6)^2} = 2.7 \times 10^{-11} W / m^2$$


Solution:
a) The maximum and minimum distances are for a line separation of $s = 1 \text{ mm}$ are:

Minimum: $\Delta \theta = \frac{s}{L} = \frac{1.22\lambda}{D}$, implies $L = 3.0 \text{ m}$ and Maximum: $L = 10.4 \text{ m}$.
b) The pupil diameter will be $D = \frac{1.22 \lambda L}{s} = 6.71 \times 10^{-4} L$