Problem 1

Consider Eq. (15.2) in the textbook. Substituting in $a_n$ and $b_n$ from Eq. (15.7) and (15.8) to derive a closure relation satisfied by sin’s and con’s. Hint: the right-hand side of the equation must be a delta function $\delta(x - x')$.

Problem 2

Using residue theorem, calculate

$$
\int_0^\infty \frac{dx}{(x^2 + a^2)^2}
$$

Note that the residue of a function $f(z)$ at a point $z_0$ is defined as the coefficient of $1/(z - z_0)$ when $f(z)$ is expanded as $\ldots + a_{-2}/(z - z_0)^2 + a_{-1}/(z - z_0) + a_0 + a_1(z - z_0) + \ldots$.

Problem 3

Evaluate the following definite integral

$$
I = \int_0^{2\pi} \frac{d\theta}{1 + \epsilon \cos \theta}
$$

where $|\epsilon| < 1$, by letting $z = e^{i\theta}$ and $\cos \theta = (z + z^{-1})/2$ and choosing a contour $|z| = 1$, a unit circle.

Problem 4

Using the Fourier transformation and contour integral, solve for the Green’s function equation,

$$
m \frac{dG(t, t')}{dt} + \beta G(t, t') = \delta(t - t')
$$

Problem 5

Solve Green’s function in Problem 4 by considering the equation in the region $t < t'$ and $t > t'$ separately. Crossing $t = t'$, use the discontinuity condition for $G$ obtained by integrating the equation from $t' - \epsilon$ to $t' + \epsilon$. 