Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

**Problem 1** In calculating the Green’s function for the Schrodinger equation in 3D, we encounter the following 3D integral,

\[
I(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3k e^{i\vec{k} \cdot \vec{r}} e^{\frac{i\hbar \vec{k} \cdot \vec{r}}{2m}}
\]  \hspace{1cm} (1)

Evaluate the above integral as a product of three one-dimensional integrals.

**Problem 2**

Consider the one-dimensional Green’s function equation for \( G \),

\[
\frac{\partial G(t, t')}{\partial t} + \beta G(t, t') = \delta(t - t')
\]  \hspace{1cm} (2)

Consider solving \( G \) with two different boundary conditions. And show that the difference of the two Green’s functions is a solution of the homogenous equation. [Hint: consider \( G \) vanishes before \( t = t' \) and after \( t = t' \) separately.]

**Problem 3**

Consider the Helmholtz equation in 2D

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + k^2 \psi = 0
\]  \hspace{1cm} (3)

in a rectangular box with length \( L_x \) in the x-direction and \( L_y \) in the y-direction. Therefore, \( x \) and \( y \) coordinates are limited to \( (0, L_x) \) and \( (0, L_y) \), respectively. If \( \psi \) vanishes at the boundary of the box, calculate the normal modes for the system.

**Problem 4**

Consider the normal modes of a drum in Sec. 20.2. Show that the different normal modes are orthogonal to each other. Using the trick that I have shown you in the lecture.

**Problem 5**

Derive a completeness relation for the normal modes on a drum by expanding a function \( f(r, \theta) \) in terms of the normal modes.