Physics 374 Homework 9

1) In class we showed a general argument why the lowest nonvanishing multipole coefficient was independent of the origin. We also showed explicitly for the case where the m=0 multipole was absent that the m=1 multipole coefficient was independent of the origin. The explicit calculation was done by shifting the origin in the integral of the m=1 moment. In this problem I would like you to use the same method to show explicitly for the case where the m=0 and m=1 multipole coefficients vanish the m=2 multipole coefficient is independent of the origin.

2) Consider a cylindrical surface of radius R. Suppose the potential on the surface of the cylinder is fixed to the following value: \( \Phi(r, \vartheta) = V_0 \left( \sin^2(\vartheta) - \frac{1}{2} \right) \) and that there are no charges away from the surface of the cylinder.
   a) Find the value of \( \Phi(r, \vartheta) \) for \( r < R \) (Hint only one multipole will contribute)
   b) Find the electric field for \( r < R \).
   c) Find the value of \( \Phi(r, \vartheta) \) for \( r > R \)

3) Consider two cylindrical surface of radii \( R_1 \) and \( R_2 \). Suppose the potential on the surface of the two cylinders are fixed to the following values:
   \( \Phi(R_1, \vartheta) = V_1 \left( \sin^2(\vartheta) - \frac{1}{2} \right) \) and \( \Phi(R_2, \vartheta) = V_1 \left( \cos^2(\vartheta) - \frac{1}{2} \right) \); there are no charges away from the surfaces of the cylinders.
   Find the value of \( \Phi(r, \vartheta) \) for \( R_1 < r < R_2 \) (Hint only one multipole will contribute)

4) Consider the following set up. There is vacuum in the region \( |x^2 - y^2| \leq R^2 \) where \( R \) is a fixed parameter. The region \( |x^2 - y^2| > R^2 \) is filled with conducting material which can be held fixed potentials Note that there are four distinct conducting regions---in polar coordinates they are: i) \( -\frac{\pi}{4} < \vartheta < \frac{\pi}{4} \); ii) \( \frac{\pi}{4} < \vartheta < \frac{3\pi}{4} \); iii) \( \frac{3\pi}{4} < \vartheta < \frac{5\pi}{4} \); and iv) \( \frac{5\pi}{4} < \vartheta < \frac{7\pi}{4} \). Suppose that regions i) and iii) are held at a potential of \( +V_0 \) while regions ii) and iv) are held at \( -V_0 \). Find the potential for the region \( |x^2 - y^2| \leq R^2 \). This involves solving the Laplace equation subject to a boundary condition. Hint: You should be able to convince yourself that only the m=\pm 2 terms will contribute.