1. A cylinder of radius R contains charge according to the following charge density:
\[ \rho(x) = c x^2 \delta(R-r) \]
where \( c \) is a constant, \( r = \sqrt{x^2 + y^2} \) and \( \delta \) is the usual step function. The goal of this problem is to find the electric field for the region \( r>R \).

a) As a first step calculate the multipole coefficients. You should find only the \( m=0 \) and \( m=2 \) terms contribute. Hint: write the \( x^2 \) term in polar coordinates.

b) Find the potential \( \Phi \). As the \( m=0 \) term is nonzero you must pick an arbitrary \( r_0 \).

c) Find an expression for the electric field.

2. For the problem in 1 find the potential for \( r<R \). Hint: use the general solution.

3. Consider 4 line charges oriented along the z axis, 2 with a charge per unit length of \(+\lambda\) located at \((x,y)=(d,d)\) and \((x,y)=(-d,-d)\) and 2 with a charge per unit length of \(-\lambda\) located at \((x,y)=(d,-d)\) and \((x,y)=(-d,d)\).

a) Show that the exact expression for the potential is given by
\[ \Phi(\vec{r}) = -2 \lambda \log \left( \frac{\sqrt{(r^2 + 2d^2 + 2\sqrt{2}r d \cos(\phi - \frac{\pi}{4}))} (r^2 + 2d^2 + 2\sqrt{2}r d \cos(\phi - \frac{3\pi}{4}))}{\sqrt{(r^2 + 2d^2 + 2\sqrt{2}r d \cos(\phi - \frac{\pi}{4})) (r^2 + 2d^2 + 2\sqrt{2}r d \cos(\phi - \frac{3\pi}{4}))}} \right) \]
Hint: Use the alternative general expression and note that the integral just gets replaced by a discrete sum over the line charges. Recall \( |\vec{x} - \vec{y}| = \sqrt{x^2 + y^2 + 2\vec{x} \cdot \vec{y}} \)

b) Show that the multipole coefficients are given by
\[ b_o = 0 \]
\[ a_m = \left( \frac{\sqrt{2}d}{m} \right)^m (\exp(i \frac{m\pi}{4}) + \exp(i \frac{m3\pi}{4}) - \exp(i \frac{m\pi}{4}) - \exp(i \frac{m7\pi}{4})) \]
Hint: Again note that that the integral just gets replaced by a discrete sum over the line charges.

c) Write an approximate expression for the potential based on the multipole expansion and including up to the \( m=4 \) term.

d) Test how well the multipole expression works by using Mathematica to plot the exact expression, and multipole truncated at \( m=2,3 \) and \( 4 \) as a function of \( r \) for \( \phi=\pi/4 \).

4. It was argued in class that multipole coefficient with smallest nonvanishing \( m \) is independent of the choice of origin. Show that the \( m=2 \) multipole for the problem does not change if move the origin to the position of the charge at the lower left.