PHYS 373 (Fall 2015):
Mathematical Methods for Physics II
Homeworks

Note: the 1st homework is simply signing the honor pledge (but still it is *compulsory*); the actual homework starts with #2.
And, please sign the honor pledge on a *separate* sheet of paper from HW 2.

1 Homework 1 (due Friday, September 11 in folder outside Room 3118 of PSC by 5 pm.)

Print your name and sign (with date) the Honor Pledge (which covers *all* assignments, including exams):

“I pledge on my honor that I will not give or receive any unauthorized assistance (including from other persons and online sources) on all examinations and homework assignments in this course.”

General guidelines for homework

(1). Please read the problem *number* (when it is from the textbook by Boas) correctly.
In particular, note that the number to left of decimal point refers to *section* of the corresponding chapter and the one to the *right* is the actual problem number.

(2). Many of the following homework problems have *multiple* parts. So, it is your responsibility to read the statement of the problem carefully.

(3). Since many (or even most) of the problems here are (purely) mathematical ones, the entire idea here is to actually do all the algebra and calculus *by hand*. So, *unless it is (explicitly) stated otherwise*, try *not* to use any computer programs (such as mathematica) for this purpose or to look up (purely) mathematical formulae in reference tables (including online).

(4). You are welcome to ask for help (for example, hints) on homework from the instructor or TA. Also, “limited” discussion with other students *is* allowed (and encouraged): for example, just in order to get started, but the actual problem-solving part should be your own work. And, you should try *not* to get any *other* outside help.

(5). In order to get full credit for homework (*and* exam) problems, you should show as many steps as you can.
2 Homework 2 (due Friday, September 11 in folder outside Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 7 of the textbook Boas.

2.1 Basic Fourier Series
Problems (i) 5.2 and (ii) 5.8

2.2 Complex form of Fourier Series
Problem (i) 7.13

2.3 Using other intervals for Fourier Series
Problems (i) 8.15; (ii) 8.16 and (iii) 8.20

2.4 Even and Odd Functions
Problems (i) 9.5 and (ii) 9.10

3 Homework 3 (due Thursday, September 17 in lecture or in folder outside Room 3118 of PSC by 5 pm.)

All problems in this homework are from the textbook Boas.

3.1 Parseval’s theorem for Fourier series
Problem 11.2 from chapter 7.

3.2 Application of Parseval’s theorem
Problem 11.5 from chapter 7.

3.3 Basics of Fourier integral
Problems (i) 12.3 and (ii) 12.17 from chapter 7.

3.4 Application of Fourier integral
Problem 12.23 from chapter 7.

For 2nd part of this problem, you might wish to see the comment about the “jump” in Example 2 of this section.
3.5 Parseval’s theorem for Fourier integral
Problem 12.31 from chapter 7.

Note: the remaining problems in this homework are from chapter 3 of Boas.

3.6 Gram-Schmidt method for “usual” vectors
Problem 10.4.
(Think of this as “warm-up” for problem 14.16 below.)

3.7 General vector spaces
Problems (i) 14.5 and (ii) 14.9
Note: the set might not be a vector space!

3.8 Gram-Schmidt method for general vector space
Problem 14.16
This problem is basically about completing the “missing” steps from Example 6 of this section, which was briefly discussed in class (but you might have to read the rest on your own). You do not have to solve actually problem 14.15 for this purpose, i.e., you can simply assume that integral of an odd power of \( x \) from \( x = -1 \) to \( x = 1 \) vanishes.

These are called Legendre polynomials: we will return to them when we solve differential equations using power series as our next topic.

4 Homework 4 (due Thursday, September 24 in lecture or in folder outside Room 3118 of PSC by 5 pm.)
All problems in this homework are from chapter 12 of the textbook Boas.

4.1 Comparing series solutions vs. elementary/standard method, i.e., using chapter 8
Problems

(i) 1.1 (Hint: review section 2 of chapter 8);
(ii) 1.4 (case in Eq. 5.16 onwards from section 5 of chapter 8);
(iii) 1.6 (case in Eq. 5.12-5.15 from section 5 of chapter 8) and
(iv) 1.7 [case (d) from section 7 of chapter 8]
Note: you are supposed to solve above differential equations using both elementary and power series methods.

Also, Problem 1.7 looks similar to Legendre’s equation, but while the former can be solved by a standard/elementary method, try doing so for the latter (cf. the series solution that we will eventually end up using)!

4.2 Explicit expressions for low-order Legendre polynomials

Problem 2.1

Note: there is a typo in the statement of the problem: it should be $P_l (evaluated \ at \ x = 1) = 1.$

See also HW 3.8, i.e., problem 14.16 (and example 6 of that section) from chapter 3.

5 Homework 5 (due Thursday, October 1 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 12 of the textbook Boas.

5.1 Using Rodrigues’ formula for Legendre polynomials

Problems (i) 4.2 and (ii) 4.4

[The 2nd problem above, i.e., # 4.4 from chapter 12 of Boas, was used in lecture in (i) (one way of) proving orthogonality of Legendre polynomials and then for (ii) computing the normalization of Legendre polynomials.]

5.2 Recursion relations for Legendre polynomials

Problems (i) 5.4; (ii) 5.5; (iii) 5.6 and (iv) 5.7

Note: the first problem above, i.e., proving 2nd recursion relation in Eq. 5.8 (b) from Boas, does make use of generating function for Legendre polynomials. However, the other problems (i.e., proving rest of recursion relations) rely only on manipulating first two recursion relations, i.e., there is no need to use generating function here.

5.3 Using the associated differential equation to show orthogonality of sine/cosine

Problem 7.1
5.4 Legendre series

Problems (i) 9.1 (Hint: feel free to make use of result of example 9.1 just above) and (ii) 9.4

Note: in the very last problem (# 9.4 from chapter 12 of Boas), try to do integrals yourself first, but if you really are not able to, then you can look them up in tables.

6 Homework 6 (due Thursday, October 8 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from the textbook Boas.

6.1 One more Legendre series!

Problem 9.2 from chapter 12.

The associated Legendre functions will be used as the basis for other functions when working in spherical coordinates. So, it is useful to learn/know their properties, which is the topic of the next four problems below.

6.2 Orthogonality of associated Legendre functions

Problem 10.3 from chapter 12.

Note: as mentioned in the statement of the problem, the method to be used here (i.e., based directly on the functions being solutions to differential equation) is similar to what was done in section 7/lecture (for proving orthogonality of Legendre functions) and problem 7.1 in homework 5 (for sine/cosine functions).

6.3 Normalization of associated Legendre functions

Problems (i) 10.7; (ii) 10.9 and (iii) 10.10 from chapter 12.

Note: The first two problems above, i.e., # 10.7 and 10.9 are used, in turn, to compute normalization in 3rd problem (# 10.10).

6.4 Simplifying products/ratios of Gamma (Γ) functions

Problems (i) 3.2 and (ii) 3.7 from chapter 11.

Note: Γ functions will be useful when we study Bessel functions (which are solutions to Bessel’s equation).
6.5 Bessel functions for half-integer order can be related to trigonometric functions

Problem 12.9 from chapter 12.

7 Homework 7 (due Thursday, October 22 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 12 of the textbook Boas.

7.1 Second solution of Bessel’s equation for integer order

Problem 13.2

Hint: as mentioned in lecture (and discussed below Eq. 13.1 of Boas), for integer order \( p \), the first few terms in series in Eq. 12.9 are zero because \( \Gamma(n - p + 1) \) in denominator is infinite. In this way, you have to show that \( J_{-p} \) starts with the term \( x^p \) (how this happens was already indicated in lecture). In addition, you need to relate subsequent terms in the two series.

7.2 Recursion relations for Bessel functions

Problems 15.1, 15.3 and 15.5

[Note that for solving problem 15.3, you can simply assume the recursion relations given in Eqs. 15.1 and 15.2: of course the first one was proved in lecture (and in Boas) and you are supposed to show (in the previous HW problem, i.e., # 15.1) the 2nd one using a similar method.]

7.3 Other equations with solutions being related to Bessel functions

Problem 16.7

Hint: you have to find appropriate \( a, b \) and \( c \) in Eq. 16.1 so that it reduces to the one given in problem.

7.4 Normalization of Bessel functions

Problem 19.1.

7.5 Using normalization of Bessel function to compute definite integral

Problem 19.2.
8  Homework 8 (due Thursday, October 29 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 13 of the textbook Boas.

8.1 Laplace’s equation for semi-infinite plate, but varying temperature on bottom plate

Problems (i) 2.1 and (ii) 2.2

[Note: in the 2nd problem above, the width of plate is 20 cm. (vs. 10 cm. in the 1st one and in the example done in book (Fig. 2.1)/lecture).]

8.2 Laplace’s equation for finite plate

Problem 2.12

[Hint: see discussion around Fig. 2.2 of Boas (also done in lecture).]

8.3 Diffusion equation with final temperature at two ends being zero

Problems (i) 3.2 and (ii) 3.4

Note: answer to the 1st problem above is given so that the entire credit here is for the derivation!

And, a clarification for the 2nd problem above: initially, the entire 1st slab is at 0° (similarly, 2nd one is at 20°). Then, at t = 0, the outer surfaces of the “combined” slab are kept at 0°.

9  Homework 9 (due Thursday, November 5 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 13 of the textbook Boas.

9.1 Sketching normal modes for circular membrane

Problem 6.1 (a) (only), i.e., there no need to submit (although you can still try it!) part (b), which is animations!

Note: just show rough sketches, like in Fig. 6.1: there is no need to show numerical values of k, etc. here.
9.2 Numerical values of fundamental frequencies for circular membrane

Problem 6.2

9.3 Normal modes of a rectangular membrane

Problem 6.3, but there is no need to do 2nd part on square membrane (although you are welcome to try it!).

Note: Sketch for the first four modes (at least), (again) just roughly, and write the formula for displacement under each sketch (like in Fig. 6.1 and HW 9.3 above).

9.4 Laplace’s equation in spherical coordinates: steady-state temperature inside a sphere

Problems (i) 7.1 and (ii) 7.10 from chapter 13.

Note: You can follow the example done in section 7 (which was also done in lecture), except that the temperature on the surface of the sphere is different than the one given there. And, this boundary condition in first problem does not have $\phi$-dependence, but the 2nd one does.

10 Homework 10 (due Thursday, November 12 in lecture or in folder near Room 3118 of PSC by 5 pm.)

All problems in this homework are from the textbook Boas.

The problems at the beginning are from chapter 13.

10.1 Diffusion equation with final temperatures on two faces being non-zero

Problems (i) 3.5 and (ii) 3.6

[Hint: see example 2 (from section 3) in Boas (also done in lecture).]

10.2 Insulated boundaries: Neumann problem

Problem 3.7

Note: see discussion after example 2 in Boas (also done in lecture). In addition, you have to think/read more for this problem (i.e., it is not simply ”plug and chug”), as mentioned in the hints and comments given after the statement of problem. In particular, there is a connection of this problem to an earlier on Laplace’s equation (#2.15 from Boas) and to example 2 from Boas on diffusion equation.
10.3  Schroedinger equation

Problem 3.12

(You can print out the plot and attach it to your HW.)

The remaining problems are from chapter 14.

10.4  Real and imaginary parts of function: using Cauchy-Reimman conditions (Theorem II)

Problems (i) 1.6; (ii) 2.6, (iii) 1.7 and (ii) 2.7

Note: the first two problems above involve the same function; similarly the other two.

10.5  Using Theorem III

Problems (i) 2.37 and (ii) 2.38

Note: See comments in Boas just above these problems and example worked out around Fig. 2.4 (which was also done in lecture).

10.6  Using Theorem IV

Problem 2.60

Note: You can follow example 4 (or the one done in lecture).

11  Homework 11 (due Tuesday, November 24 in lecture or in folder outside Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 14 of the textbook Boas.

As usual, read the statements of the problems carefully (including presence of more than one part and guidelines/suggestions for solving).

11.1  Contour integrals and Cauchy’s results

Problem (i) 3.3 and (ii) 3.17

11.2  Laurent series

Problems (i) 4.4 and (ii) 4.10
11.3 How to find residues
Problems (i) 6.1 and (ii) 6.14 and 6.14′
(Note that the meaning of prime is explained just below problem 35.)

12 Homework 12 (due Thursday, December 3 in lecture or in folder outside Room 3118 of PSC by 5 pm.)

All problems in this homework are from chapter 14 of the textbook Boas.

12.1 (Continuation of) Finding residues
Problems (i) 6.21 and 6.21′ and (ii) 6.31 and 6.31′

12.2 Complexifying real integrals
Problems (i) 7.7; (ii) 7.13 and (iii) 7.17
(You should be able to follow the methods from examples 1-3 in these problems.)

13 Homework 13 (due Thursday, December 10 in lecture or in folder outside Room 3118 of PSC by 5 pm.)

13.1 (Continuation of) Evaluating definite integrals using residue theorem
Problems (i) 7.26; (ii) 7.33 and (iii) 7.36
[Follow Examples 4 and 5 (also done in lecture).]

13.2 Locating roots using Argument Principle
Problem 7.46
[Hint: you can follow Example 6 (also done in lecture).]

13.3 Contour integral using using Argument Principle
Problem 7.52
13.4 Nature of point at infinity
Problem 8.10

13.5 Residue at infinity
Problem 8.15