Solutions to Homework 1

February 11, 2003

Problem 1

a: Angular frequency \( \omega = \sqrt{\frac{\kappa}{\text{mass}}} \), \( \kappa = 800 \text{N/m} \), mass = 2kg, therefore, \( \omega = 20 \text{rad/s} \).

b: Block released from rest \( A=20 \text{cm}. \) displacement \( y = 20 \cos(\omega t) \), velocity \( \nu = \frac{dy}{dt} \), Acceleration \( a = \frac{d\nu}{dt} = -\omega^2 y \). For downward positive \( y = 12 \text{cm} \), \( a = -4800 \text{cm/s}^2 \), upward negative \( y = -12 \text{cm} \), \( a = 4800 \text{cm/s}^2 \); With Energy conservation, \( E = \frac{1}{2} \kappa A^2 = \frac{1}{2} m \nu^2 + \frac{1}{2} \kappa y^2 \). we have \( \nu = \pm \omega \sqrt{A^2 - y^2} \), for \( y = \pm 12 \text{cm}, \nu = \pm 320 \text{cm/s} \)

Problem 2

a Consider what happens when the mass is given a displacement \( x > 0 \), one spring will be stretched \( x \) and the other will be compressed \( x \), they will each exert a force of magnitude \( 20 \text{N/m} \times x \) on the mass in the direction opposite to the displacement. Hence the total restoring force is \( F = -20 \times x - 20 \times x = -40 \text{N/m} \times x \), \( F = -\kappa x \) tell us the system has a spring constant \( \kappa = 40 \text{N/m} \). Hence, the period \( T = 2\pi \sqrt{\frac{\text{mass}}{\kappa}} = 0.54 \text{s} \).

b: when the mass is displaced a distance \( y \) downward, each spring is stretched a distance \( y \). The net restoring force is then \( F = 2 \times (-20 \text{N/m})y \). Hence, again, from \( F = -\kappa x \), we have \( \kappa = 40 \text{N/m} \), the same as in (a). \( T = 0.54 \text{s} \).

Problem 3

Mass \( m_2 \) shoots off when the spring stretched maximally and carrying kinetic energy \( K \) away from the system, so the amplitude of oscillation of \( m_1 \) satisfies \( 1/2 \kappa A^2 = 1/2 m_1 \nu^2 \), where \( \nu \) is the velocity at the equilibrium position.
To find \( \nu \), potential \( U \) of spring = maximum \( K_t \) of masses.

\[
\kappa d^2/2 = (m_1 + m_2)\nu^2, \quad \text{giving } \nu^2 = (\kappa d^2)/(m_1 + m_2).
\]

Then we have \( 1/2\kappa A^2 = 1/2m_1\nu^2 = 1/2(\kappa d^2)/(m_1 + m_2) \), then \( A = d\sqrt{m_1/(m_1 + m_2)} \).

**Problem 4**

a: If the stick is rotated through a small angle \( \theta \), each spring is stretched a distance \( L\theta/2 \). Each spring causes a torque= \( \theta/2 \times L/2 \) with both torque in the same direction. The torque equation is

\[
-2\kappa \theta (L/2)(L/2) = I_{cm} \alpha
\]

where \( I_{cm} = mL^2/12 \), the momentum to center of mass; \( \alpha = d^2\theta/d^2t \)

then \( \alpha = -6\kappa/m)\theta \). This is the equation for harmonic motion.

b) Frequency \( f = \sqrt{6\kappa/m}/2\Pi \).

c) The velocity reach the maxium when the stick passes the horizontal. Let \( \theta_0 \) be the initial angle, so the maximum velocity = \( (L/2)(2\Pi f)\theta_0 = L\theta_0(1.5\kappa/m)^{1/2} \).

**Problem H1**

Equation of motion is \( \tau = I\alpha \), where \( \tau \) is the external torque, \( I \) is the momentum of inertia, \( \alpha \) is the angular acceleration, and \( \tau \) and \( I \) are about to the pivot point.

The contribution to \( \tau \) is due to the rod and disk

\[
\tau = -mg(L/2)\sin\theta - Mg(R + L)\sin\theta
\]

where \( \theta \) is a small angular displacement from the vertical.

\[
I = I_{rod} + I_{disk} = (mL^2)/3 + [(MR^2)/2 + M(R + L)^2]
\]

where we used the parallel axis law to calculate \( I_{disk} \).

For small \( \theta \), \( \sin\theta \approx \theta \), so the equation of the motion is

\[
-g[M R + M L + (m L)/2] \theta = [mL^2/3 + 3MR^2/2 + 2MRL + ML^2] \alpha
\]

so easily to get the period \( f = 1/T, (2\Pi f)^2 \) is the coefficient of the \( \theta \) term.

\[
T^2 = [4\Pi^2/g][mL^2/3 + (3MR^2)/2 + 2MRL + ML^2]/[ML + MR + (mL)/2]
\]