as shown below:

This is the difference of two squares and can be expanded

\[ (k + \rho a - \omega_0)^2 - (k - \rho a - \omega_0)^2 = 0. \]

Therefore, the system, in expanded form, the characteristic equation for

\[ 0 = \begin{vmatrix} \omega_0 & k + \rho a - \omega_0 & k \omega_0 - \omega_1 \\ k + \rho a - \omega_0 & k^2 + \rho \omega_2 - \omega_1 & k \omega_1 - \omega_2 \\ k & \omega_1 & \omega_2 \end{vmatrix} \]

Zero must equal zero for \( a \) or \( b \) to have a value other than

In both cases the determinant in the denominator

\[ \begin{vmatrix} \omega_0 & k + \rho a - \omega_0 & k \omega_0 - \omega_1 \\ k + \rho a - \omega_0 & k^2 + \rho \omega_2 - \omega_1 & k \omega_1 - \omega_2 \\ 0 & k & \omega_2 \end{vmatrix} = 0 \]

Similarly for \( b \).

By Kramer's rule,

\[ \begin{vmatrix} \omega_0 & k + \rho a - \omega_0 & k \omega_0 - \omega_1 \\ k + \rho a - \omega_0 & k^2 + \rho \omega_2 - \omega_1 & k \omega_1 - \omega_2 \\ 0 & k & \omega_2 \end{vmatrix} = \begin{vmatrix} \omega_0 & k + \rho a - \omega_0 & k \omega_0 - \omega_1 \\ k + \rho a - \omega_0 & k^2 + \rho \omega_2 - \omega_1 & k \omega_1 - \omega_2 \\ 0 & k \end{vmatrix} = \begin{vmatrix} k^2 + \rho \omega_2 - \omega_1 & k \omega_1 - \omega_2 \\ k \omega_1 - \omega_2 & \omega_2 \end{vmatrix} = k \omega_1 - \omega_2 \]

Putting these equations in matrix form,

\[ 0 = \begin{bmatrix} k \omega_1 - \omega_2 & \omega_1 \omega_2 \end{bmatrix} \begin{bmatrix} a \rho + \rho \omega_2 - \omega_1 & 0 \\ 0 & a \rho + \rho \omega_2 - \omega_1 \end{bmatrix} \]

(1)

Since \( \omega_2 \neq 0 \), divide both equations by \( \omega_2 \).