Problem for Honors:

1. The time it takes the particle to travel a distance $dx$ is
   
   $$ dt = \frac{dx}{v}, \quad v = \frac{dx}{dt}. $$

   The period of an oscillation with total energy $E$ and amplitude $x_m$ is
   
   $$ T = 4 \int_0^{x_{m(t)}} dt' = 4 \int_0^{x_m} \frac{dx}{v} $$

   From energy conservation $\frac{1}{2}mu^2 + V = E = \text{constant} = \frac{1}{2}kx_m^2 + a x_m^4$

   $$ v = \frac{\sqrt{2E - V}}{m} $$

   $$ T = 4 \int_0^{x_m} \frac{dx}{\sqrt{\frac{2E - V}{m}}} = 4 \int_0^{x_m} \frac{dx}{\sqrt{E - V}} $$

2. For $a = 0, \quad V(x) = \frac{1}{2}kx^2, \quad E = \frac{1}{2}kx_m^2$

   (let $m = 1, \quad k = 1$ and $x_m = 1$

   $$ T = 4 \int_0^1 \frac{dx}{\sqrt{E - V}} = 4 \int_0^1 \frac{dx}{\sqrt{\frac{1}{2} - x^2}} $$

   Let $x = \cos \theta, \quad dx = -\sin \theta d\theta \Rightarrow -\sin \theta d\theta = -\sin \theta d\theta$

   $$ T = 4 \int_{\frac{x_m}{2}}^{\frac{x_m}{2}} -\sin \theta d\theta = 2\pi$$

   Dimensional analysis: let's see $V(x) = \frac{1}{2}kx^2$

   $k$ unit is $\frac{\text{Energy}}{\text{Distance}^2}$.