Problem 1. [10 points]

a) Solve the differential equation $y'' + 4y = 0$ for the following initial conditions
   - $y(0) = 0$, $y(\pi/4) = 1$
   - $y(\pi/2) = -1$, $y'(\pi/2) = 1$
   - $y(0) = 0$, $y'(0) = 1$
   - $y(\pi/4) = a$, $y''(0) = b$

b) Solve the differential equation $y'' + y = 2y'$ with the boundary conditions $y(0) = 1$, $y(1) = 1$

Problem 2. [10 points]

Solve the following homogeneous differential equations with an exponential ansatz and determine the most general solution. In case that the solution is complex, determine the corresponding real solution
   - $2y'' - 12y' + 10y = 0$
   - $4y'' - 12y' + 9y = 0$
   - $y'' + 2y' + 5y = 0$

Problem 3. [10 points]

A mass $m$ is subject to a resistive force $F = -bv$, where $b$ is a constant and $v$ is the velocity but no restoring force.

a) Show, that the displacement as a function of time takes the form $x = C - \frac{v_0}{\gamma} e^{-\gamma t}$, where $\gamma = \frac{b}{m}$. To show this, use the method used in class to solve the damped oscillator. That is, assume the displacement as a function of time takes the form $x(t) = e^{\alpha t}$, plug into the equation of motion and solve for possible values of $\alpha$. Check, if you have the right number of free constants in your solution.

b) At $t = 0$ the mass is at rest at $x = 0$. At this instant a driving force $F = F_0 \cos(\omega t)$ is switched on. Find the values of $A$ and $\delta$ in the steady state solution $x(t) = A \cos(\omega t - \delta)$.

c) Write down the general solution (the sum of parts (a) and (b) of this problem) and find the values of $C$ and $v_0$ from the initial conditions that $x = 0$ and $x' = 0$ at $t = 0$. Sketch $x$ as a function of $t$.

[10 points]