1. [10 pts] A long wire carrying a 10A current is charging a large circular parallel plate capacitor with radius ‘d’ as shown below.

a. [3 pts] Derive an expression for the magnetic field a distance r from the wire where r<d (at point P1).

At the point P1 there is no electric field hence we apply amperes law by constructing an Amperian circular loop passing through P1 with radius r having the axis along the wire. Using symmetry the Magnetic field lines are along the tangent of the loop as shown in the figure. Then using amperes law \( \oint B \cdot dl = \mu_0 I \), we have \( B = \frac{\mu_0 I}{2\pi r} \).

b. [7 pts] Derive an expression for the magnetic field midway between the parallel plate capacitors at a distance ‘r’ from the center where r<d (at point P2).

Solution:

We might assume a circular loop passing through P2 with radius ‘r’ and axis coinciding with the straight wire. Then one might show that the B field is tangential to the loop and uniform according to symmetry as shown in the figure. Then we might use one of the maxwells equations
\[
\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_s + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]
where inside the capacitor \( I_s = 0 \)

therefore
\[
\oint B \cdot dl = \mu_0 \frac{\varepsilon_0 \frac{\partial (\mathbf{E} \cdot \mathbf{A})}{\partial t}}{D A} = \mu_0 \frac{\varepsilon_0 A \frac{\partial V_C}{\partial t}}{D} = \mu_0 \frac{\varepsilon_0 A \frac{\partial Q}{\partial t}}{C D} = \mu_0 \frac{\varepsilon_0 A \frac{\partial Q}{D C}}{\partial t} = \mu_0 \frac{\varepsilon_0 A}{D} D I
\]
Where we have used the formula for capacitance of a parallel plate capacitor of plate separation $D$ and plate area $A$ (which is constant over time),

$$C = \frac{\varepsilon_0 A}{D} = \frac{\varepsilon_0 \pi d^2}{D}$$

and $Q = CV_c$

And the relationship between electrostatic field and electrostatic potential,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{V_c}{D}$$

where $x$ is the direction normal to the plates and the potential gradient is uniform between the plates and we have neglected the negative sign since we are dealing with magnitudes only.

Therefore we get,

$$2\pi r B = \mu_0 A \frac{A}{r^2} I = \mu_0 \frac{r^2}{2\pi d^2} I$$

which implies that $B = \mu_0 \frac{r}{2\pi d^2} I$ in the clockwise direction as seen from the left end of the wire.