Quiz #11d

[10 pts] Consider a hydrogen atom in an orbital angular momentum state of \( l=3 \).

a. [5 pts] What is the minimum angle between \( L \) and the z-axis?

b. [5 pts] What is the maximum angle between \( L \) and the z-axis?

For \( l=3 \) the total angular momentum is \( L=\sqrt{l(l+1)}\hbar=\sqrt{(3(3+1))}\hbar=\sqrt{12}\hbar \)

the z-component of the angular momentum given by,
\[
L_\cos\theta=L_z=m\hbar
\]

where \( m \) is an integer varying from \(-3\) to \(+3\) hence there are 7 possibilities as illustrated by the diagram.

a) Therefore the minimum angle between the \( L \) and the \( z \) axis is given by
\[
\theta=\cos^{-1}(m\hbar/\sqrt{12}\hbar) \quad \text{the minimum value being for maximum value of } m=3 \quad \text{which gives} \quad \theta_{\min}=\cos^{-1}(3/\sqrt{12})=30^0
\]

b) The maximum value of theta is given by \( (|m|)_{\min}=0 \), therefore \( \theta_{\max}=90^0 \)