Solution to Quiz 10

The wave-function of a particle in infinite potential well, is given by

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) & 0 \leq x \leq L \\ 0 & x < 0 \text{ or } x > L \end{cases}$$

Here, \( n = 1, 2, \ldots \) etc. For \( n = 3 \)

$$\psi_3(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{3 \pi x}{L} \right)$$

as \( x \) goes from 0 to \( L \),

$$\frac{3 \pi x}{L}$$

goes from 0 to \( 3 \pi \).

Hence the wave-function is simply a sine function, plotted from 0 to \( 3 \pi \), with amplitude \( \sqrt{\frac{2}{L}} \).

Hence \( \psi(x) \) & \( |\psi(x)|^2 \) will look like

The particle is most likely to be found where \( |\psi(x)|^2 \) has a maxima, or wherever \( \psi(x) \) has a maxima or minima, i.e. extrema.
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Since $\sin$ function has an extrema at $(n+\frac{1}{2})\pi$,
the particle is most likely to be found at

$$\frac{3x\pi}{L} = (n + \frac{1}{2})\pi$$ \hspace{1cm} 0 \leq x \leq L

$$\Rightarrow x = \frac{L}{3}(n + \frac{1}{2})$$

$n=0$, $x = \frac{L}{6}$; $n=1$, $x = \frac{L}{2}$; $n=2$, $x = \frac{5L}{6}$

For $n=3,4,5...$ etc we get $x > L$ where
we know there is no wavefunction.

Hence the positions where the particle is most likely
to be is

$$x = \frac{L}{6}, \frac{3L}{6} = \frac{L}{2}, \frac{5L}{6}$$

\[\text{Diagram}\]