CONCEPTUAL QUESTIONS

25.1. $\theta_1$ decreases. As the crystal is compressed, the spacing $d$ between the planes of atoms decreases. The Bragg condition is $m\lambda = 2d\cos\theta_m$ so as $d$ decreases, $\cos\theta_m$ must increase. But $\cos\theta$ increases as $\theta$ decreases.

25.2. (a) $E_a > E_b > E_c$ because the energy per photon depends only on the frequency so $E = hf = hc/\lambda$. The smaller wavelengths correspond to higher frequencies. (b) $N_c > N_b > N_a$ because the powers are equal, there must be more photons when the energy per photon is less.

25.3. $\frac{E_2}{E_1} = \frac{hc/\lambda_2}{hc/\lambda_1} = \frac{\lambda_1}{2\lambda_1} = \frac{1}{2}$

25.5. Fast electrons will have a shorter wavelength leading to less diffraction spreading and better resolution.

25.7. Because $E_a = n^2 \frac{h^2}{8mL^2}$ we see that for a given $n$, $E_a$ is inversely proportional to $L^2$. If $L$ is doubled then $E_a$ is decreased by a factor of 4. So the new $E_i = 1 \times 10^{-19}$ J.

25.8. It is the same, or $1.0 \times 10^{-20}$ J. $E_{H_1} = \frac{h^2}{8m_0L_0^2}$ $E_{H_2} = \frac{h^2}{8(4m_0)\left(\frac{L_0}{2}\right)^2} = E_{H_1}$

EXERCISES AND PROBLEMS

25.4. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition. Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where $m = 1, 2, 3, \ldots$ For first and second order diffraction,

$$2d\cos\theta_1 = (1)\lambda \quad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\frac{\cos\theta_2}{\cos\theta_1} = 2 \Rightarrow \theta_2 = \cos^{-1}\left(\frac{2\cos\theta_1}{2}\right) = \cos^{-1}\left(\frac{2\cos68^\circ}{2}\right) = 41^\circ$$
25.7 Model: The angles corresponding to the various diffraction orders satisfy the Bragg condition.
Solve: The Bragg condition is \(2d \cos \theta_n = m\lambda\), where \(m = 1, 2, 3, \ldots\). The maximum possible value of \(m\) is the number of possible diffraction orders. The maximum value of \(\cos \theta_n\) is 1. Thus,

\[
2d = m\lambda \Rightarrow m = \frac{2d}{\lambda} = \frac{2(0.180 \text{ nm})}{(0.085 \text{ nm})} = 4.2
\]

We can observe up to the fourth diffraction order.

25.10 Model: Use the photon model of light.
Solve: The energy of a photon with wavelength \(\lambda\) is

\[
E = \frac{hc}{\lambda}
\]

Similarly, \(E_2 = \frac{hc}{\lambda_2}\). Since \(E_2\) is equal to \(2E_1\),

\[
\frac{hc}{\lambda_2} = 2 \frac{hc}{\lambda_1} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{1}{2} = \frac{600 \text{ nm}}{2} = 300 \text{ nm}
\]

Assess: A photon with \(\lambda = 300 \text{ nm}\) has twice the energy of a photon with \(\lambda = 600 \text{ nm}\). This is an expected result, because energy is inversely proportional to the wavelength.

25.12 Solve: Your mass is, say, \(m = 70 \text{ kg}\) and your velocity is 1 m/s. Thus, your momentum is \(p = mv = (70 \text{ kg})(1 \text{ m/s}) = 70 \text{ kg m/s}\). Your de Broglie wavelength is

\[
\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ Js}}{70 \text{ kg m/s}} = 9 \times 10^{-36} \text{ m}
\]

25.14 Visualize: We’ll employ Equations 25.8 \((\lambda = h/p)\) and 25.9 \((E = p^2/2m)\) to express the wavelength in terms of kinetic energy.
Solve: First solve Equation 25.9 for \(p\): \(p = \sqrt{2mE}\).

\[
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34} \text{ J s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(2.4 \times 10^{-19} \text{ J})}} = 1.0 \text{ nm}
\]

Assess: The energy given is about 1.5 eV, which is a reasonable amount of energy. The resulting wavelength is a few to a few dozen times the size of an atom.

25.18 Model: Model the 5.0-fm-diameter nucleus as a box of length \(L = 5.0\) fm.
Solve: The proton’s energy is restricted to the discrete values

\[
E_n = \frac{\hbar^2 n^2}{8mL^2} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right)^2 n^2}{8(1.67 \times 10^{-27} \text{ kg})(5.0 \times 10^{-15} \text{ m})} = \left(1.316 \times 10^{-12} \text{ J}\right)n^2
\]

where \(n = 1, 2, 3, \ldots\). For \(n = 1\), \(E_1 = 1.3 \times 10^{-12} \text{ J}\), for \(n = 2\), \(E_2 = \left(1.316 \times 10^{-12} \text{ J}\right)4 = 5.3 \times 10^{-12} \text{ J}\), and for \(n = 3\), \(E_3 = 9E_1 = 1.2 \times 10^{-11} \text{ J}\).
25.21. **Model:** Use the photon model of light.  
**Solve:** (a) The wavelength is calculated as follows:

\[ \lambda = \frac{hc}{E} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right) \left(3.0 \times 10^8 \text{ m/s}\right)}{1.0 \times 10^{-13} \text{ J}} = 2.0 \times 10^{-12} \text{ m} \]

(b) The energy of a visible-light photon of wavelength 500 nm is

\[ E_{\text{visible}} = h\left(\frac{c}{\lambda}\right) = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right) \left(3.0 \times 10^8 \text{ m/s}\right)}{500 \times 10^{-9} \text{ m}} = 3.978 \times 10^{-19} \text{ J} \]

The number of photons \( n \) such that \( E_{\gamma} = nE_{\text{visible}} \) is

\[ n = \frac{E_{\gamma}}{E_{\text{visible}}} = \frac{1.0 \times 10^{-13} \text{ J}}{3.978 \times 10^{-19} \text{ J}} = 2.5 \times 10^5 \]

25.22. **Model:** Use the photon model.  
**Solve:** The energy of a 1000 kHz photon is

\[ E_{\text{photon}} = hf = \left(6.63 \times 10^{-34} \text{ Js}\right) \left(1000 \times 10^3 \text{ Hz}\right) = 6.63 \times 10^{-28} \text{ J} \]

The energy transmitted each second is \( 20 \times 10^9 \text{ J} \). The number of photons transmitted each second is \( 20 \times 10^9 \text{ J/6.63 \times 10^{-28} J} = 3.0 \times 10^{11} \).  

25.25. **Model:** Use the photon model of light and the Bragg condition for diffraction.  
**Solve:** The Bragg condition for the reflection of x-rays from a crystal is \( 2d \cos \theta_m = m \lambda \). To determine the angles of incidence \( \theta_m \), we need to first calculate \( \lambda \). The wavelength is related to the photon’s energy as \( E = \frac{hc}{\lambda} \). Thus,

\[ \lambda = \frac{hc}{E} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right) \left(3.0 \times 10^8 \text{ m/s}\right)}{1.50 \times 10^{-15} \text{ J}} = 1.326 \times 10^{-10} \text{ m} \]

From the Bragg condition,

\[ \theta_m = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left[\frac{1.326 \times 10^{-10} \text{ m}}{2 \left(0.21 \times 10^{-9} \text{ m}\right)}\right] = \cos^{-1}(0.3157) \Rightarrow \theta_1 = \cos^{-1}(0.3157) = 71.6^\circ \]

Likewise, \( \theta_2 = \cos^{-1}(0.3157 \times 2) = 50.8^\circ \) and \( \theta_3 = 18.7^\circ \). Note that \( \theta_4 = \cos^{-1}(0.3157 \times 4) \) is not allowed because the \( \cos \theta \) cannot be larger than 1. Thus, the x-rays will be diffracted at angles of incidence equal to \( 18.7^\circ \), \( 50.8^\circ \), and \( 71.6^\circ \).
25.29. **Model:** Particles have a de Broglie wavelength given by \( \lambda = h/p \). The wave nature of the particles causes an interference pattern in a double-slit apparatus.

**Solve:** (a) Since the speed of the neutron and electron are the same, the neutron’s momentum is

\[
p_n = m_n v_n = \frac{m_n}{m_e} m_e v_n = \frac{m_n}{m_e} p_e
\]

where \( m_n \) and \( m_e \) are the neutron’s and electron’s masses. The de Broglie wavelength for the neutron is

\[
\lambda_n = \frac{h}{p_n} = \frac{h}{p_e} = \lambda_e \frac{m_e}{m_n}
\]

From Section 22.2 on double-slit interference, the fringe spacing is \( \Delta y = \lambda L/d \). Thus, the fringe spacing for the electron and neutron are related by

\[
\Delta y_n = \frac{\lambda_n}{\lambda_e} \Delta y_e = \frac{m_e}{m_n} \Delta y_e = \left( \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) \left( 1.5 \times 10^{-3} \text{ m} \right) = 8.18 \times 10^{-7} \text{ m} = 0.818 \mu \text{ m}
\]

(b) If the fringe spacing has to be the same for the neutrons and the electrons,

\[
\Delta y_e = \Delta y_n \Rightarrow \lambda_e = \lambda_n \Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_n v_n} \Rightarrow v_e = v_n \frac{m_n}{m_e} = \left( 2.0 \times 10^6 \text{ m/s} \right) \left( \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) = 1.1 \times 10^3 \text{ m/s}
\]

25.30. **Model:** Electrons have a de Broglie wavelength given by \( \lambda = h/p \). The wave nature of the electrons causes a diffraction pattern.

**Solve:** The width of the central maximum of a single-slit diffraction pattern is given by Equation 22.22:

\[
w = \frac{2L}{a} \frac{2L}{ap} = \frac{2L}{amv} = \frac{2 \left( 1.0 \text{ m} \right) \left( 6.63 \times 10^{-34} \text{ Js} \right)}{\left( 1.0 \times 10^{-6} \text{ m} \right) \left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 1.5 \times 10^6 \text{ m/s} \right)} = 9.7 \times 10^{-4} \text{ m} = 0.97 \text{ mm}
\]

25.32. **Model:** Electrons have a de Broglie wavelength given by \( \lambda = h/p \).

**Visualize:** Please refer to Figure 25.11. [REFER TO THE SUPPLEMENTAL FOR THE IMAGE AND ANALYSIS THEREOF]

Notice that a scattering angle \( \theta = 60^\circ \) corresponds to an angle of incidence \( \phi = 30^\circ \).

**Solve:** Equation 25.6 describes the Davisson-Germer experiment: \( D \sin \left( \frac{\phi}{2} \right) = m \lambda \). Assuming \( m = 1 \), this equation simplifies to \( D \sin 2\theta = \lambda \). Using \( \lambda = h/mv \), we have

\[
D = \frac{h}{mv \sin 2\theta} = \frac{6.63 \times 10^{-34} \text{ Js}}{\left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 4.30 \times 10^6 \text{ m/s} \right) \sin (60^\circ)} = 1.95 \times 10^{-10} \text{ m} = 0.195 \text{ nm}
\]
25.33. **Model:** A confined particle can have only discrete values of energy.

**Solve:**
(a) Equation 25.14 simplifies to

\[ E_n = \frac{\hbar^2}{8mL} n^2 = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right)^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)\left(0.70 \times 10^{-3} \text{ m}\right)^2} = \left(1.231 \times 10^{-10} \text{ J}\right) n^2 \]

Thus, \( E_1 = \left(1.231 \times 10^{-10} \text{ J}\right) \left(1^2\right) = 1.2 \times 10^{-10} \text{ J}, \ E_2 = \left(1.231 \times 10^{-10} \text{ J}\right) \left(2^2\right) = 4.9 \times 10^{-10} \text{ J}, \) and \( E_3 = 1.1 \times 10^{-10} \text{ J}. \)

(b) The energy is \( E_2 - E_1 = 4.9 \times 10^{-10} \text{ J} - 1.2 \times 10^{-10} \text{ J} = 3.7 \times 10^{-10} \text{ J}. \)

(c) Because energy is conserved, the photon will carry an energy of \( E_2 - E_1 = 3.69 \times 10^{-10} \text{ J}. \) That is,

\[ E_2 - E_1 = E_{\text{photon}} = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_2 - E_1} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right)\left(3.0 \times 10^8 \text{ m/s}\right)}{3.69 \times 10^{-10} \text{ J}} = 540 \text{ nm} \]

25.34. **Model:** A particle confined in a one-dimensional box has discrete energy levels.

**Solve:**
(a) Equation 24.14 for the \( n = 1 \) state is

\[ E_1 = \frac{\hbar^2}{8mL} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right)^2}{8\left(10 \times 10^{-3} \text{ kg}\right)\left(0.10 \text{ m}\right)^2} = 5.5 \times 10^{-44} \text{ J} \]

The minimum energy of the Ping-Pong ball is \( E_1 = 5.5 \times 10^{-44} \text{ J}. \)

(b) The speed is calculated as follows:

\[ E_1 = 5.50 \times 10^{-44} \text{ J} = \frac{1}{2}mv^2 = \frac{1}{2}\left(10 \times 10^{-3} \text{ kg}\right)v^2 \Rightarrow v = \sqrt{\frac{2(5.50 \times 10^{-44} \text{ J})}{10 \times 10^{-3} \text{ kg}}} = 3.3 \times 10^{-31} \text{ m/s} \]

25.39. **Model:** A particle confined in a one-dimensional box of length \( L \) has the discrete energy levels given by Equation 24.14.

**Solve:**
(a) Since the energy is entirely kinetic energy,

\[ E_n = \frac{\hbar^2}{8mL^2} n^2 = \frac{p^2}{2m} = \frac{1}{2}mv_n^2 \Rightarrow v_n = \frac{\hbar}{2mL} n \quad n = 1, 2, 3, \ldots \]

(b) The first allowed velocity is

\[ v_1 = \frac{6.63 \times 10^{-34} \text{ Js}}{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(0.20 \times 10^{-3} \text{ m}\right)} = 1.82 \times 10^6 \text{ m/s} \]

For \( n = 2 \) and \( n = 3 \), \( v_2 = 3.64 \times 10^6 \text{ m/s} \) and \( v_3 = 5.46 \times 10^6 \text{ m/s} \).