P30.1 \[ B = \frac{\mu_0 I}{2R} = \frac{\mu_0 (\sqrt{2} \pi R)}{2R} = 12.5 \text{ T} \]

P30.7 For the straight sections \( dB \times \hat{z} = 0 \). The quarter circle makes one-fourth the field of a full loop:
\[ B = \frac{\mu_0 I}{4 \cdot 2R} = \frac{\mu_0 I}{8R} \text{ into the paper} \]
\[ B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(5.00 \text{ A})}{8(0.030 \text{ m})} = 26.2 \mu \text{T into the paper} \]

*P30.11 (a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the \((-\hat{k})\) field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate \( y = -|y| \). Here the total field is
\[ B = \frac{\mu_0 I}{2\pi r} \hat{k} + \frac{\mu_0 I}{2\pi r} \hat{-k} \]
\[ 0 = \frac{\mu_0}{2\pi} \left[ \frac{50 \text{ A}}{|y| + 0.28 \text{ m}} (-\hat{k}) + \frac{30 \text{ A}}{|y|} \hat{k} \right] \]
\[ 50|y| = 30(|y| + 0.28 \text{ m}) \]
\[ 50(-y) = 30(0.28 \text{ m} - y) \]
\[ -20y = 30(0.28 \text{ m}) \text{ at } y = -0.420 \text{ m} \]

(b) At \( y = 0.1 \text{ m} \) the total field is \( B = \frac{\mu_0 I}{2\pi r} \hat{k} + \frac{\mu_0 I}{2\pi r} \hat{-k} \):
\[ B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \left( \frac{50 \text{ A}}{(0.28 - 0.10) \text{ m}} (-\hat{k}) + \frac{30 \text{ A}}{0.10 \text{ m}} (-\hat{k}) \right) = 1.16 \times 10^{-4} \text{ T} (-\hat{k}) \]

The force on the particle is
\[ \mathbf{F} = q\mathbf{v} \times \mathbf{B} = \left(-2 \times 10^{-6} \text{ C}\right) \left(150 \times 10^6 \text{ m/s}\right) \left(1.16 \times 10^{-4} \text{ N} \cdot \text{C} \cdot \text{m}^{-1}\right) \left(-\mathbf{k}\right) = 3.47 \times 10^{-2} \text{ N}\left(-\mathbf{j}\right) \]

(c) We require \[ \mathbf{F}_e = 3.47 \times 10^{-2} \text{ N} \left(+\mathbf{j}\right) = q\mathbf{E} = (-2 \times 10^{-6} \text{ C})\mathbf{E}. \]

So \[ \mathbf{E} = \left[-1.73 \times 10^4 \mathbf{j} \text{ N/C}\right]. \]

*P30.19* Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2.

(a) For the equilibrium of wire 3 we have

\[ R_{1 \text{ on } 3} = R_{2 \text{ on } 3} \]

\[ 1.5(20 \text{ cm} + d) = 4d \]

\[ d = \frac{30 \text{ cm}}{2.5} = 12 \text{ cm to the left of wire 1} \]

(b) For the equilibrium of wire 1,

\[ \frac{\mu_0 I_3 (1.5 \text{ A})}{2\pi (12 \text{ cm})} = \frac{\mu_0 (4 \text{ A})(15 \text{ A})}{2\pi (20 \text{ cm})} \]

\[ I_3 = \frac{12}{20} \text{ A} = \frac{3}{5} \text{ A down} \]

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

P30.22 Let the current \( I \) be to the right. It creates a field \( B = \frac{\mu_0 I}{2\pi d} \) at the proton’s location. And we have a balance between the weight of the proton and the magnetic force

\[ mg(-\mathbf{j}) + q\mathbf{v}(-\mathbf{j}) \times \frac{\mu_0 I}{2\pi d} \mathbf{k} = 0 \text{ at a distance } d \text{ from the wire} \]

\[ d = \frac{q\mu_0 I}{2\pi mg} = \frac{\left[1.60 \times 10^{-19} \text{ C}\right] \left[2.30 \times 10^4 \text{ m/s}\right] \left[4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right] \left[1.20 \times 10^{-6} \text{ A}\right]}{2\pi \left[1.67 \times 10^{-27} \text{ kg}\right] \left[9.80 \text{ m/s}^2\right]} = 5.40 \text{ cm} \]
P30.36 (a) \( \Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \) where \( A \) is the cross-sectional area of the solenoid.

\[
\Phi_B = \left( \frac{\mu_0 NI}{\ell} \right) \left( \pi R^2 \right) = 7.40 \mu \text{Wb}
\]

(b) \( \Phi_B = \mathbf{B} \cdot \mathbf{A} = BA = \left( \frac{\mu_0 NI}{\ell} \right) \left[ \pi \left( R^2 - r^2 \right) \right] \)

\[
\Phi_B = \left[ \frac{4\pi \times 10^{-7} \ T \cdot \text{m/A}}{(0.300 \ m)} \right] \left( \pi \left( 8.00^2 - 4.00^2 \right) \right) \left( 10^{-3} \ m \right)^2 = 2.27 \mu \text{Wb}
\]

P30.38 \( \frac{d\Phi_B}{dt} = \frac{d\mathbf{E}}{dt} = \frac{dQ}{dt} = \frac{I}{\varepsilon_0} \)

(a) \( \frac{d\mathbf{E}}{dt} = \frac{I}{\varepsilon_0} A = 7.19 \times 10^{11} \ \text{V/m} \cdot \text{s} \)

(b) \( \int B \cdot ds = \varepsilon_0 \mu_0 \frac{d\Phi_B}{dt} \) so \( 2\pi rB = \varepsilon_0 \mu_0 \left[ \frac{Q}{\varepsilon_0 A} \cdot \pi R^2 \right] \)

\[
B = \frac{\mu_0 B_0}{2A} \left( \frac{0.200 \left( 5.00 \times 10^{-2} \right)}{2\pi (0.100)^2} \right) = 2.00 \times 10^{-7} \ \text{T}
\]

P30.40 \( B = \mu_0 nI = \frac{N}{2\pi R} I \) so \( I = \frac{(2\pi R)B}{\mu_0 N} = \frac{2\pi (0.100 \ m) (1.30 \ T)}{5000 (4\pi \times 10^{-7} \ \text{Wb/A} \cdot \text{m})(470)} = 277 \ \text{mA} \)

P30.42 \( C = \frac{\frac{I R}{B}}{B} = \frac{(4.00 \ \text{K})(10.0\%)(8.00 \times 10^{27} \ \text{atom g}^{-1} \text{m}^{-3})(5.00)(9.27 \times 10^{-24} \ \text{g}^{-1} \text{m}^{-2})}{5.00 \ \text{T}} = 2.97 \times 10^4 \ \text{K} \cdot \text{J} \cdot \text{T}^{-2} \cdot \text{m}^3 \)

P30.46 (a) \( B_h = B_{\text{max}} = \frac{\mu_0 nI}{2R} = \frac{(4\pi \times 10^{-7})(5.00)(0.500)}{0.300} = 12.6 \ \mu \text{T} \)

(b) \( B_h \sin \phi \rightarrow B = \frac{B_h}{\sin \phi} = \frac{12.6 \ \mu \text{T}}{\sin 13.0^\circ} = 56.6 \ \mu \text{T} \)

FIG. P30.46
P30.50 Suppose you have two 100-W headlights running from a 12-V battery, with the whole 200 W 12 V = 17 A current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so μ = μ₀. Model the current as straight. Then,

\[
B = \frac{\mu_0 I}{2\pi r} = \left(\frac{4\pi \times 10^{-7}}{2\pi(0.6)}\right)^{17} \approx 10^{-5} \text{ T}.
\]

If the local geomagnetic field is \(5 \times 10^{-5} \text{ T}\), this is \(\approx \) 10⁻¹ times as large, enough to affect the compass noticeably.

*P30.58* From example 30.6, the upper sheet creates field \(B = \frac{\mu_0 J_z}{2} \hat{k}\) above it and \(\frac{\mu_0 J_z}{2} (-\hat{k})\) below it. Consider a patch of the sheet of width \(w\) parallel to the \(z\) axis and length \(d\) parallel to the \(x\) axis. The charge on it \(\sigma w d\) passes a point in time \(\frac{dw}{v}\), so the current it constitutes is \(\frac{q}{t} = \frac{\sigma w dv}{d} = \sigma v\) and the linear current density is \(J_z = \frac{\sigma w v}{w} = \sigma v\). Then the magnitude of the magnetic field created by the upper sheet is \(\frac{1}{2} \mu_0 \sigma v\). Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field \(\frac{1}{2} \mu_0 \sigma (-v)\) above it and \(\frac{1}{2} \mu_0 \sigma v\) below it.

(a) Between the plates, their fields add to \(\mu_0 \sigma v \text{ away from you horizontally}\).

(b) Above both sheets and below both, their equal-magnitude fields add to zero.

(c) The upper plate exerts no force on itself. The field of the lower plate, \(\frac{1}{2} \mu_0 \sigma (-v)\) will exert a force on the current in the \(w\)-by-\(d\)-section, given by

\[
\mathcal{F} \times B = \sigma w v d \left(\frac{1}{2} \mu_0 \sigma (-v)\right) = \frac{1}{2} \mu_0 \sigma^2 v^2 w d.
\]

The force per area is \(\frac{1}{2} \mu_0 \sigma^2 v^2 \text{ up}\).  

\[\text{ANSWER KEY - Page 18}\]
(d) The electrical force on our section of the upper plate is
\[ E_{\text{elec}} = \frac{\sigma \ell w}{2 \varepsilon_0} \, (-\hat{z}) = -\frac{\ell w \sigma^2}{2 \varepsilon_0} \, (-\hat{z}). \]

The electrical force per area is
\[ \frac{\ell w \sigma^2}{2 \varepsilon_0} \, \hat{d} \text{ow} \hat{n} = \frac{\sigma^2}{2 \varepsilon_0} \, \hat{d} \text{ow} \hat{n}. \]
To have \( \frac{1}{2} \mu_0 \sigma^2 v^2 = \frac{\sigma^2}{2 \varepsilon_0} \), we require

\[ v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = -\frac{1}{\sqrt{4\pi \times 10^{-7} \text{ (Tm/A)} \, (N/\text{TA m}) \, 8.85 \times 10^{-12} \text{ (C}^2/\text{N m}^2) \, (\text{A s/C})^2}} = 3.00 \times 10^8 \text{ m/s}. \]

This is the speed of light, not a possible speed for a metal plate.