Solutions to hw 8

\[ \Delta \phi = \frac{2\pi}{\lambda} (r_2 - r_1) \]

\[ r_1^2 = x^2 + \left(y - \frac{d}{2}\right)^2 \]

\[ r_2^2 = x^2 + \left(y + \frac{d}{2}\right)^2 \]

my numbers: \( x = 1000 \text{ m}, \; d = 272 \text{ m} \)

\[ \Delta y = 400 \text{ m} \implies r_2 - r_1 = 100.2 \text{ m} \]

\[ \Delta \phi = 4\pi \implies \lambda = 50.2 \text{ m} \]

b) The next minimum occurs when \( \Delta \phi = 5\pi \implies r_2 - r_1 = \frac{5}{2} \lambda \). The algebra needed to obtain our exact solution for \( y \) is tedious, but we can obtain a good approximation by writing \( y = y_0 + \Delta y \) and expanding with respect to \( \Delta y \), such that

\[ r_2 = \left[ x^2 + \left(y_0 + \Delta y \pm \frac{d}{2}\right)^2 \right]^{1/2} = r_0^{(0)} + \frac{3\Delta y}{2} + \frac{3\Delta y^2}{4} \Delta y = r_0^{(0)} + \frac{y_0}{r_0^{(0)}} \Delta y + \frac{3\Delta y^2}{4} \]

\[ \Delta (r_2 - r_1) \approx \left( \frac{y_0 + \frac{d}{2}}{r_0^{(0)}} - \frac{y_0 - \frac{d}{2}}{r_0^{(0)}} \right) \Delta y \]

where \( r_0^{(0)} \) indicates the preceding results of part a). Using

\[ y_0 = 400 \text{ m} \implies r_0^{(0)} = 1124.6 \text{ m}, \; r_0^{(1)} = 1034.3 \text{ m} \]

\[ \Delta (r_2 - r_1) = \frac{\lambda}{2} = \left( \frac{532}{1124.6} - \frac{244}{1034.3} \right) \Delta y \implies \Delta y = 115.6 \text{ m} \]

we obtain a first approximation, but \( \Delta y \) may not be sufficiently accurate because \( \Delta y / y \) is not really small. To improve upon this approximation, we compute \( \Delta \phi \) for several nearby values and interpolate.
\[
\begin{array}{cc}
0^\circ & 0^\circ \\
116 m & 2.47 \\
120 & 2.486 \\
124 & 2.5006 \\
\end{array}
\]

Therefore, the can must travel just a little less than 124 m further to reach the next minimum.

b) Here we do use the small-angle approximation.

\[ \delta = \frac{2\pi}{\lambda} \sin \theta = \frac{v}{L} ; \ y = 1.25 \text{ cm}, L = 150 \text{ cm}, d = 0.1 \text{ mm} \Rightarrow \delta = 8.33 \times 10^{-7} \text{ m} \]

\[ \lambda = 500 \text{ nm} \Rightarrow \frac{\delta}{\lambda} = 1.67 \]

c) Intermediate

e) \[ I = I_{\text{max}} \cos \frac{2\pi}{\lambda} \Rightarrow \Delta \Phi = 2\cos^{-1}\left(\sqrt{\frac{I}{I_{\text{max}}}}\right) \]

\[ a) I_{\text{max}}/I = 0.57 \Rightarrow \delta = 1.43 \text{ rad} \text{ in minimum phase difference} \]

\[ b) \Phi = \frac{2\pi}{\lambda} \Delta \Phi \Rightarrow \Delta r = \lambda \Delta \Phi / 2\pi \; ; \; \lambda = 487.3 \text{ nm} \Rightarrow \Delta r = 111 \text{ nm} \]

d) \[ E^2 = E_1^2 + E_2^2 - 2E_1E_2 \cos \phi \]

\[ \sin \phi = \frac{E_2 - E_1}{E_1} \]

\[ \text{my numbers: } E_1 = 12, E_2 = 16.5, \phi = 60^\circ \]

\[ \therefore E = 24.8, \; \phi = 35.2^\circ \]

e) \[ n_1 = 1.47, \; n_2 = 1.33, \; t = 286 \text{ nm} \]

a) External reflection at the air-oil changes the phase by \( \pi \); but internal reflection at the oil-water interface does not change the phase.

The total phase difference between the reflected beams at normal incidence is this.
\[ \Delta \phi = 2\pi \frac{2\pi t}{\lambda} \]

Constructive interference is obtained when \( \Delta \phi = 2\pi m \) for integer \( m \), such that

\[ (m + \frac{1}{2}) \lambda = 2\pi t \]

The only solution for \( \lambda \) that lies in the visible spectrum is

\[ m = 1 \Rightarrow \lambda = 561 \text{ nm} \]

Light reflected from the air-water and then oil-air interface and then transmitted into water experiences two internal reflections that do not affect the phase. Thus,

\[ \Delta \phi = 2\pi \frac{2\pi t}{\lambda} = 2\pi m \Rightarrow m \lambda = 2\pi t \]

The only solution in the visible range is

\[ m = 2 \Rightarrow \lambda = 420 \text{ nm} \]

\[ \lambda = 580 \text{ nm}, \quad m = 1 \Rightarrow d = 290 \text{ nm} \]

\[ d = m \frac{\lambda}{2} \]
Interference between light reflected either from the top or the bottom of the air gap produces dark fringes when

\[ \phi = \frac{2\pi}{\lambda} 2t + \pi = 2\pi(m + \frac{1}{2}) \]

where \( m \) is an integer. Note that one reflection is external and the other internal, so that there is a net phase difference of \( \pi \) due to reflection. Using

\[ \lambda = 550.1 \text{ nm} \quad \pm 4 \times 10^{-3} \text{ nm} \quad \Rightarrow \quad 0 \leq m \leq 146.43. \]

Thus, including the central fringe \( (m=0) \) there are a total of 146 dark bands.

\[ 2\sqrt{h^2 + \left(\frac{d}{2}\right)^2} - d = \frac{\lambda}{2} \]

\[ h^2 = \frac{1}{4} \left(d + \frac{4h}{2}\right)^2 - \left(\frac{d}{2}\right)^2 \]

\[ = \frac{1}{16} \left(4hd + h^2\right) \]

\[ \therefore \quad h = \frac{1}{2} \sqrt{1 + 4\frac{d}{\lambda}} \approx \frac{1}{2} \sqrt{d} \]

my numbers:  \( d = 33 \text{ km} \),  \( \lambda = 345 \text{ m} \)  \( \Rightarrow \quad h = 1.64 \text{ km} \)
P37.12) If the incident light is at an angle $\theta_1$ wrt normal, the total path length difference for parallel rays has two contributions

$$\Delta s = s_2 - s_1 = d (\sin \theta_2 - \sin \theta_1)$$

Constructive interference requires

$$\Delta s = n \lambda = d (\sin \theta_2 - \sin \theta_1)$$

where $n$ is an integer.

P37.22) We can add two phases of different amplitude using the accompanying diagram. According to the law of cosines,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

but

$$\gamma = \pi - \phi \Rightarrow \cos \gamma = -\cos \phi$$

such that

$$c^2 = a^2 + b^2 + 2ab \cos \phi$$

$$\cos \theta = \frac{b \cos \phi}{c}$$

$$\Rightarrow E_0 \sin \omega t + E_{02} \sin (\omega t + \phi) = E_0 \sin (\omega t + \theta) \Rightarrow E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos \phi$$

$$\sin \theta = \frac{E_{02}}{E_0} \sin \phi$$

P37.44) Traversing the cell twice, the total phase difference between empty and full cells is

$$\Delta \phi = 2\pi \frac{n-1}{\lambda} \cdot 2L = 2\pi N \Rightarrow N = (n-1) \frac{\frac{\lambda}{2}}{\lambda} \Rightarrow n-1 = \frac{N\lambda}{2L}$$