Solution to HW 12

1) According to the Bohr model for hydrogen, the wavelength emitted in a transition between levels \( n \) and \( m \) is given by:

\[
\lambda_{nm}^{-1} = R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \quad \text{where} \quad R_H = 109737 \text{ nm}^{-1}
\]

\[
E_{nm} = \frac{\hbar c}{\lambda_{nm}} = E_0 \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \quad \text{where} \quad E_0 = 13.6 \text{ eV}
\]

The shortest wavelength and highest energy for a series with fixed \( n \) is then obtained with \( m = 0 \), which gives that:

\[
\min \lambda_n = \frac{n^2}{R_H}, \quad \max E_n = \frac{E_0}{n^2}
\]

<table>
<thead>
<tr>
<th></th>
<th>( n )</th>
<th>( \min \lambda_n )</th>
<th>( \max E_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyman</td>
<td>1</td>
<td>91.1 nm</td>
<td>13.6 eV</td>
</tr>
<tr>
<td>Balmer</td>
<td>2</td>
<td>365 nm</td>
<td>3.40</td>
</tr>
<tr>
<td>Paschen</td>
<td>3</td>
<td>850 nm</td>
<td>1.51</td>
</tr>
<tr>
<td>Bracket</td>
<td>4</td>
<td>1450 nm</td>
<td>0.85</td>
</tr>
</tbody>
</table>

2) \( E_{nm} = E_0 \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \) emitted for \( m > n \) or absorbed for \( n > m \)

A) \((m, n) = (3, 5) \Rightarrow E_{nm} = -0.071 \ E_0 \) gains the most energy

B) \((m, n) = (5, 3) \Rightarrow E_{nm} = +0.071 \ E_0 \) emits shortest wavelength

C) \((m, n) = (6, 4) \Rightarrow E_{nm} = +0.047 \ E_0 \)

D) \((m, n) = (4, 8) \Rightarrow E_{nm} = -0.047 \ E_0 \)

Losses energy for B, C; gains for A, D

3) The kinetic energy for each atom is \( K = \frac{\hbar c}{\lambda} = 10.2 \text{ eV} \) for \( \lambda = 121.6 \text{ nm} \).

Because \( K \ll mc^2 \), we can use the nonrelativistic form of energy to find \( \beta = \left( \frac{2K}{mc^2} \right)^{1/2} = 1.47 \times 10^{-4} \) where \( mc^2 = 938 \text{ MeV} \) for hydrogen. \( \Rightarrow \) \( v = \beta c = 4.42 \times 10^6 \text{ m/s} \).
4) The Bohr radius for a single-electron atom or ion is given by \( a = a_0/z \) where \( a_0 = 0.0529 \text{ nm} \) and when \( z \) is the nuclear charge in units of e.

(a) \( \text{He}^+ \Rightarrow z=2 \Rightarrow a = 0.0265 \text{ nm} \)
(b) \( \text{Li}^{2+} \Rightarrow z=3 \Rightarrow a = 0.0176 \text{ nm} \)
(c) \( \text{Be}^{3+} \Rightarrow z=4 \Rightarrow a = 0.0132 \text{ nm} \)

5) If an electron is confined to diameter \( D \) with \( \lambda \leq D \), it must have momentum \( p = \hbar/\lambda \geq \hbar/D \). For the problem \( D \) is so small and \( p \) so large that we should use relativistic kinematics with \( \gamma \gg 1 \Rightarrow K \approx pc \geq \hbar c/D \). Hence, \( D \approx 10^{-5} \text{ nm} \Rightarrow K \approx 1.2 \times 10^8 \text{ eV} \). The electric potential energy is of order \( U \approx \frac{Ze^2}{4\pi \epsilon_0} D^{-1} \sim 0.14 \text{ MeV} \) for \( z=1 \). Hence, \( K \gg U \) and we would expect the electron to escape almost immediately; it cannot be bound within a nucleus.

6) My numbers: \( m=60 \text{ kg} \), \( w=0.51 \text{ m} \), \( d=12 \text{ cm} \), \( v=10 \text{ m/s} \)
   (a) \( w \leq 10 \lambda \), \( \lambda = \frac{\hbar}{mv} \Rightarrow v \leq \frac{10 \hbar}{mw} = 1.36 \times 10^{-34} \text{ m/s} \)
   (b) \( t = \frac{d}{v} = 8.8 \times 10^{-32} \text{ s} = 2.3 \times 10^{15} \text{ times age of Universe} \)
   (c) Not in your lifetime!

7) Assume that nonrelativistic kinematics is adequate, such that \( K_{\text{max}} = (8BR)^2/2m_e \) and \( \Phi = K_{\text{max}} - \hbar c/\lambda \).
   My numbers: \( \lambda = 3.28 \text{ nm}, B = 2.1 \times 10^{-5} \text{ T}, R = 0.2 \text{ m} \)
   \( K_{\text{max}} = 1.55 \text{ eV} \), \( E_b = 3.78 \text{ eV} \Rightarrow \Phi = 2.23 \text{ eV} \)
   Note that \( K_{\text{max}} < m_e c^2 \Rightarrow \) nonrelativistic kinematics are fine.
67) Assume that nonrelativistic kinematics can be used.

\[ E = 2 \left( \frac{1}{2} m v^2 \right) - \frac{k_e e^2}{r} \]

angular momenta:
\[ L = 2 m v \left( \frac{r}{2} \right) = m vr = n \hbar \]

curvilinear motion:
\[ \frac{mv^2}{r^2} = \frac{k_e e^2}{r^2} \Rightarrow \frac{mv^2}{r} = \frac{k_e e^2}{r} \Rightarrow E = -\frac{1}{2} \frac{k_e e^2}{r} \]

\[ m \left( \frac{n \hbar}{mr} \right)^2 = \frac{1}{2} \frac{k_e e^2}{r} \Rightarrow r = 2 \frac{(n \hbar)^2}{k_e e m c^2} \]

\[ \therefore \quad E_n = -\frac{E_0}{n^2} \quad E_0 = \frac{\alpha^2 m c^2}{4} = 6.80 \text{eV} \]

\[ r_n = n^2 a_0 \quad a_0 = 2 \frac{\hbar c}{\alpha m c} = 0.106 \text{nm} \]

9) Bohr \[ E_n = -E_0 \frac{1}{n^2} \Rightarrow E_n - E_{n-1} = E_0 \frac{2n-1}{n^2(n-1)^2} = \Delta E_n \]

\[ n \to \infty \Rightarrow \Delta E_n \approx \frac{E_0}{n^4} \approx 2E_0 n^{-3} \]

classically:
\[ f = \frac{v}{2 \pi r} \quad v = \frac{n \hbar}{m \xi} \quad r = n^2 a_0 \Rightarrow f = \frac{n \hbar}{2 \pi m a_0^2 n^{-3}} \]

\[ E_0 = \frac{\alpha^2 m c^2}{2} \quad a_0 = \frac{\hbar c}{\alpha m c^2} \Rightarrow \Delta E_n = a^2 m c^2 n^{-3} \quad f = \frac{h}{n} \alpha^2 m c^2 n^{-3} \]

\[ \therefore \quad \Delta E_n = \hbar f \]

The classical orbital frequency for large \( n \) agrees with quantum mechanical transition energy, divided by \( \hbar \), for adjacent orbits.
Can a free stationary particle absorb a photon without creating one or more additional particles in the final state?

Energy and momentum conservation require

\[
E' = E_0 + mc^2 \\
p' = p_0
\]

but the invariance of rest mass requires

\[
(mc^2)^2 = E'^2 - (p'c)^2 = (E_0 + mc^2)^2 - p_0^2 c^2
\]

Finally, using \( E_0 = p_0 c \) we find that this equation requires \( E_0 = 0 \). Therefore, the process is impossible.