Assignment 6

Consider 6.0 g of helium at 40°C in the form of a cube 40 cm. on each side. Suppose 2000 J of energy are transferred to this gas.
(i) Determine the final pressure if the process is at constant volume.
(ii) What is the final volume if instead the process is at constant pressure?
(iii) On a single pV diagram, show and label both processes.

PSS 17.1: The Bermuda Triangle

Description: Find the work done on an ideal gas in a simple isochoric-isobaric-constant pV process. (PSS 17.1: Work in ideal-gas processes)

Learning Goal: To practice Problem-Solving Strategy 17.1 for problems involving calculating work done on an ideal gas.

A cylinder with initial volume $V_i$ contains a sample of gas at pressure $P_i$. The gas is heated in such a way that its pressure is directly proportional to its volume. After the gas reaches the volume $V_f$ and pressure $P_f$, it is cooled isobarically to its original
volume $V$. The gas is then cooled isochorically until it returns to the original volume and pressure.

Find the work $W$ done on the gas during the entire process.

**MODEL:** Assume that the gas is ideal and the process is quasi-static.

**VISUALIZE:** Show the process on a $pV$ diagram. Note whether it happens to be one of the basic gas processes: isochoric, isobaric, or isothermal.

**SOLVE:** Calculate the work as the area under the $pV$ curve either geometrically or by carrying out the integration:

$$\text{work done on the gas } W = -\text{(area under } pV \text{ curve)} = -\int p \, dV$$

**ASSESS:** Check your signs.

- $W > 0$ when the gas is compressed. Energy is transferred from the environment to the gas.
- $W < 0$ when the gas expands. Energy is transferred from the gas to the environment.
- No work is done if the volume doesn't change: $W = 0$.

**Model**

Start by making simplifying assumptions appropriate for the situation.

**Part A**

It is reasonable to use the *ideal-gas* model in this problem if which of the following conditions are met?

Check all that apply.

**ANSWER:**

- The temperature is well below the condensation point.
- The temperature is well above the condensation point.
- The density of the gas is low.
- The density of the gas is high.
- The pressure of the gas is much greater than atmospheric pressure.
- The pressure of the gas is much smaller than atmospheric pressure.

**Part B**
The processes involved can be assumed to be quasi-static if which of the following holds?

**ANSWER:**
- They happen slowly.
- They occur at low enough pressure.
- They occur at low enough volume.
- They occur at high enough temperature.

**Visualize**

Now draw a $pV$ diagram. Note that there are three separate processes that together form a "cycle" that returns the gas to its starting point on the diagram. Be sure to indicate the direction and type of each process on your diagram. Use your diagram to answer the following questions.

**Part C**

The work done on the gas during its expansion from volume $V$ to volume $V'$ is

**ANSWER:**
- positive
- negative
- zero

During the expansion phase described in the problem introduction, volume increases and so does the pressure. The gas performs work on the environment; hence, the work done on the gas is negative. However, the gas must be absorbing heat in this process.

**Part D**

The work done on the gas during the isobaric process is

**ANSWER:**
- positive
- negative
- zero

**Part E**

The work done on the gas during the isochoric process is

**ANSWER:**
- positive
- negative
- zero

Your $pV$ diagram should look something like the figure shown here.
Solve

Now use the information and the insights that you have accumulated to construct the necessary mathematical expressions and to derive the solution.

**Part F**

Find the mathematical expression for the amount of work $W$ done on the gas during the entire process.

**Hint F.1 How to approach the problem**

The value of the net work $W$ done on the gas during the entire process equals the sum of the amounts of work done on the gas during each step of the process.

**Part F.2 Find the work done during expansion from $V$ to $3V$**

Find $W_e$, the work done during expansion of the gas from volume $V$ to volume $3V$.

**Hint F.2.a How to compute the work**

The absolute value of the work $W_e$ done on the gas during the expansion equals the area under the curve (in our case, a segment of a straight line) corresponding to that process.

Express your answer in terms of $p$ and $V$.

**ANSWER:** $W_e = -4pV$

**Part F.3 Find the work done during isobaric compression**

Find $W_i$, the work done on the gas during isobaric compression.

**Hint F.3.a How to compute the work**
The absolute value of the work \( W \) done on the gas during the isobaric process equals the area under the curve (in our case, a segment of a straight line) corresponding to that process.

Express your answer in terms of \( p \) and \( V \).

**ANSWER:** \( W = 6pV \)

Express the work in terms of \( p \) and \( V \).

**ANSWER:** \( W = 2pV \)

Note that the value of \( W \) equals, not surprisingly, the area enclosed by the entire gas cycle as drawn on the \( pV \) diagram: \( W = A_{\text{total}} \). If the direction of the process were reversed, the overall work would have been negative: \( W = -A_{\text{total}} \).

**Assess**

When you work on a problem on your own, without the computer-provided feedback, only you can assess whether your answer seems right. The following question will help you practice the skills necessary for such an assessment.

**Part G**

If the gas cycle were reordered, so that the isochoric process came before the isobaric process, the work done on the gas during the entire new cycle would be

**Hint G.1 How to approach this question**

In the solve part of this problem, you determined mathematically that the work done on the gas was positive. However, you could also have predicted that result by noting that the expansion of the gas takes place at lower average pressure than its compression. Apply this same reasoning to predict the sign of the work in the
second process.

**ANSWER:**
- positive
- negative
- zero

This answer should make sense because the gas is compressed at a higher average pressure than that during the expansion process. Of course, it also agrees with the mathematical expression obtained before.

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**A Sliding Crate of Fruit**

**Description:** For a crate of fruit sliding down an incline with friction, calculate the work done by the friction and the temperature change of the crate if all of the heat generated is absorbed by the crate.

A crate of fruit with a mass of 37.5 kg and a specific heat capacity of 3700 J/(kg · K) slides 8.10 m down a ramp inclined at an angle of 35.2 degrees below the horizontal.

**Part A**

If the crate was at rest at the top of the incline and has a speed of 2.85 m/s at the bottom, how much work was done on the crate by friction?

**Hint A.1 How to approach the problem**

If no friction were acting, then the kinetic energy of the crate at the bottom of the incline would equal the difference in gravitational potential energy of the crate between its initial and final positions. The nonconservative (nc) frictional force is responsible for the difference. To find the work done by friction, apply energy conservation:

\[ K_i + U_i + W_{nc} = K_f + U_f \]

**Part A.2 Find the initial and final kinetic energies**

What is the kinetic energy of the crate before it starts to slide \( K_i \) and after it reaches the bottom of the ramp \( K_f \)?

Express your answer in joules as two terms separated by commas.

**ANSWER:**
\[ K_i, K_f = 0, \frac{mv^2}{2} \ J \]

**Part A.3 Find the difference between initial and final potential energy**

What is \( U_f - U_i \), the change in the potential energy of the crate from when it starts to slide to after it reaches the bottom of the ramp?

**Part A.3.a A helpful formula and a helpful diagram**

The difference in the potential energy of the crate is given by

\[ U_f - U_i = -mgh \]
where \( \bar{h} \) is the height shown in the figure below. What is the value of \( \bar{h} \)?

Express your answer in meters.
ANSWER: \[ \bar{h} = \frac{L \sin(\theta)}{\cos(\theta)} \text{ m} \]

Express your answer in joules.
ANSWER: \[ U_f - U_i = -mgL \sin(\theta) \text{ J} \]

Use 9.81 \( \text{m/s}^2 \) for the acceleration due to gravity and express your answer in joules.
ANSWER: \[ W_f = \frac{mg^2}{2} - (mgL \sin(\theta)) \text{ J} \]

The frictional force opposes the motion of the crate, so the work done on the crate by friction must be a negative quantity.

\[ \text{Part B} \]

If an amount of heat equal to the magnitude of the work done by friction is absorbed by the crate of fruit and the fruit reaches a uniform final temperature, what is its temperature change \( \Delta T \)?

\[ \text{Hint B.1 Equation for temperature change} \]

The quantity of heat needed to increase the temperature of an object by a certain amount is given by \( Q = mc\Delta T \), where \( m \) is the object's mass, \( c \) is its specific heat, and \( \Delta T \) is the temperature change (in kelvins) of the object. In this case \( Q \) is a positive quantity since the temperature of the crate is increasing.

ANSWER: \[ \Delta T = \frac{gL \sin(\theta)}{c} - \frac{\theta}{2c} \text{ K} \]
Of course, the assumptions of "total heat absorption" and "uniform temperature change" are not very realistic; still, this simplified model provides a useful reminder about the transformation of mechanical energy into thermal energy when nonconservative forces are present.

**Melting Ice**

**Description:** Ice, initially below freezing, is heated at a constant rate. Calculate the time elapsed before the ice starts to melt, and the time elapsed before the temperature rises above 0 degrees Celsius.

An open container holds ice of mass 0.535 kg at a temperature of -19.1 °C. The mass of the container can be ignored. Heat is supplied to the container at the constant rate of \(890 \text{ J/minute}\).

The specific heat of ice is \(2100 \text{ J/kg - K}\) and the heat of fusion for ice is \(334 \times 10^6 \text{ J/kg}\).

**Part A**

How much time \(t_{\text{start}}\) passes before the ice starts to melt?

**Hint A.1  How to approach the problem**

Calculate the heat needed to raise the temperature of the ice to its melting point, and use this to find the time elapsed given the heating rate.

**Part A.2  Calculate the heat needed**

Calculate the heat \(Q\) needed to raise the temperature of the ice to the melting point.

**Hint A.2.a  Equation for heat gain without phase change**

Until the ice begins to melt at 0 °C, the equation \(Q = mc\Delta T\) can be used to calculate the heat added to the ice.

**ANSWER:** \(Q = m2100T\) J

**ANSWER:** \(t_{\text{start}} = \frac{m2100T}{k}\) minutes

**Part B**

From the time when the heating begins, how much time \(t_{\text{rise}}\) does it take before the temperature begins to rise above 0 °C?

**Hint B.1  How to approach the problem**

Calculate the heat needed to melt the ice and combine this with the rate that heat is
being added to the system to find the time it will take to melt the ice.

**Part B.2 Calculate the time to melt the ice**

Starting when the ice reaches 0°C, how much time \( t_{melt} \) is needed until the ice completely melts?

**Hint B.2.a Heat needed to melt the ice**

The heat needed to melt a mass \( m = 0.535 \text{ kg} \) of a material is given by \( Q = mL_f \), where \( L_f \) is the heat of fusion for the material.

**ANSWER:**

\[
\frac{m \cdot 334 \cdot 10^3}{L_f} \hspace{1cm} \text{minutes}
\]

Remember that the question is asking for the time elapsed from when the heating first begins, not from the time the ice begins melting.

**ANSWER:**

\[
\frac{m \cdot 334 \cdot 10^3}{L_f} + \frac{m \cdot 210000}{L_f} \hspace{1cm} \text{minutes}
\]

**Problem 17.61**

**Description:** \( n \) of a monatomic gas follows the process shown in the figure. (a) How much heat energy is transferred to or from the gas during process 1 \( \rightarrow \) 2? (b) How much heat energy is transferred to or from the gas during process 2 \( \rightarrow \) 3? (c)...

0.120 \( \text{mol} \) of a monatomic gas follows the process shown in the figure.

**Part A**

How much heat energy is transferred to or from the gas during process 1 \( \rightarrow \) 2?
Problem 17.29

**Description:** A gas cylinder holds $n$ of $\text{O}_2$ at $T$ and a pressure of $p$. The gas expands adiabatically until the volume is doubled. What are the final (a) pressure and (b) temperature? (a) ... (b) ...

A gas cylinder holds 0.380 mol of $\text{O}_2$ at 180 °C and a pressure of 3.20 atm. The gas
expands adiabatically until the volume is doubled. What are the final (a) pressure and (b) temperature?

Part A

**ANSWER:** \[ P \left( \frac{1}{2} \right) \] atm

Part B

**ANSWER:** \[ T \left( \frac{1}{2} \right) - 273 \] °C

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**Problem 17.27**

**Description:** A container holds m of oxygen at a pressure of p. (a) How much heat is required to increase the temperature by T at constant pressure? (b) How much will the temperature increase if this amount of heat energy is transferred to the gas at constant...

A container holds 1.30 \( \frac{\text{m}}{} \) of oxygen at a pressure of 9.00 atm.

**Part A**

How much heat is required to increase the temperature by 120 °C at constant pressure?

**ANSWER:** \[ \square \frac{29.2T}{32} \] J

**Part B**

How much will the temperature increase if this amount of heat energy is transferred to the gas at constant volume?

**ANSWER:** \[ \frac{29.2T}{20.3} \] °C
Problem 17.37

Description: m1 of aluminum at 200 degree(s) C and m2 of copper are dropped into V of ethyl alcohol at 15 degree(s) C. The temperature quickly comes to T2. (a) What was the initial temperature of the copper?

12.0 g of aluminum at 200 °C and 19.0 g of copper are dropped into 55.0 cm³ of ethyl alcohol at 15 °C. The temperature quickly comes to 28.0 °C.

Part A

What was the initial temperature of the copper?

ANSWER: \[ \frac{m_1 \cdot (T_2 - 473) + T_0 \cdot V / 298 (T_2 - 288) + T_2 - 273}{m_2} °C \]

Two Conventions, One Law

Description: Several conceptual and computational questions focusing on relationships between U, Q and W. Questions emphasize that W, depending on the convention used by the problem authors, may mean either work done on or by the system.

Learning Goal: To understand the first law of thermodynamics, written using either "work done on the system" or "work done by the system."
is called heat. Heat transfer is very common in various processes, and it is important to extend the law of conservation of energy to those processes that involve not just mechanical work but also heat. Such an "extended" version of the law of conservation of energy is known as the first law of thermodynamics, and it can be expressed as

\[ \Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = Q + W, \]

where \( \Delta E_{\text{sys}} \) is the change in the system's total energy, which is the sum of the change in the system's mechanical energy, \( \Delta E_{\text{mech}} \), and the change in the system's internal energy, \( \Delta E_{\text{th}} \) (assuming that all changes in internal energy are associated only with thermal energy); \( Q \) is the amount of heat transferred to the system; and \( W \) is the amount of work done on the system. Note the italicized words "to" and "on." These words are short, yet important: They contain, in effect, the sign convention; that is, they help you choose a positive or negative sign for the quantities that enter your calculations. For instance, a positive value of heat indicates energy being transferred from the environment to the system. Similarly, when work is done on the system, energy is transferred to the system and \( W > 0 \).

We will usually assume that no changes in mechanical energy occur in thermodynamic processes; that is, \( \Delta E_{\text{mech}} = 0 \), so the previous equation can be rewritten as

\[ \Delta E_{\text{sys}} = \Delta E_{\text{th}} = Q + W. \]

The first law of thermodynamics can be written in more than one way. If, instead of the work done on the system, we consider the work done by the system on the environment, then energy is transferred from the system to the environment and the system's total energy decreases. Thus,

\[ \Delta E_{\text{sys}} = \Delta E_{\text{th}} = Q - W, \]

where \( W \) now represents the work done by the system. When using this form for the first law, remember that \( W > 0 \) when work is done by the system.

There is no particular advantage in using one form over the other. When you solve problems, be guided by your common sense. Ask yourself "Does energy increase or decrease as a result of this process?" And remember that change is always "the final quantity" minus "the initial quantity"—that is, the change in a certain physical quantity is positive when the quantity increases its value and negative when the quantity decreases its value.

Part A
A system dissipates 12 J of heat into the surroundings; meanwhile, 28 J of work is done on the system. What is the change of the internal energy \(\Delta E_{\text{int}}\) of the system?

ANSWER:  
- 40 J  
- 16 J  
16 J  
40 J

Part B

A system absorbs 12 J of heat from the surroundings; meanwhile, 28 J of work is done on the system. What is the change of the internal energy \(\Delta E_{\text{int}}\) of the system?

ANSWER:  
- 40 J  
- 16 J  
16 J  
40 J

Part C

A system absorbs 12 J of heat from the surroundings; meanwhile, 28 J of work is done by the system. What is the change of the internal energy \(\Delta E_{\text{int}}\) of the system?

ANSWER:  
- 40 J  
- 16 J  
16 J  
40 J

Part D

A system experiences a change in internal energy of -36 kJ in a process that involves a transfer of 14 kJ of heat into the surroundings. Simultaneously, which of the following is true?

ANSWER:  
22 kJ of work is done by the system.  
22 kJ of work is done on the system.  
50 kJ of work is done by the system.  
50 kJ of work is done on the system.

Part E

A system experiences a change in internal energy of 36 kJ in a process that involves a transfer of 14 kJ of heat into the surroundings. Simultaneously, which of the
following is true?

ANSWER:  
- 22 kJ of work is done by the system.
- 22 kJ of work is done on the system.
- 50 kJ of work is done by the system.
- 50 kJ of work is done on the system.

Part F

A system experiences a change in internal energy of 14 kJ in a process that involves a transfer of 36 kJ of heat into the system. Simultaneously, which of the following is true?

ANSWER:  
- 22 kJ of work is done by the system.
- 22 kJ of work is done on the system.
- 50 kJ of work is done by the system.
- 50 kJ of work is done on the system.

Part G

In a certain process, the energy change of the system is 250 kJ. The process involves 480 kJ of work done by the system. Find the amount of heat $Q$ transferred in this process.

Express your answer numerically in kilojoules. Make your answer positive if the heat is transferred into the system; make it negative if the heat is transferred into the surroundings.

ANSWER:  
$Q = 730$ kJ

Part H

In a certain process, the energy change of the system is 250 kJ. The process involves 480 kJ of work done on the system. Find the amount of heat $Q$ transferred in this process.

Express your answer numerically in kilojoules. Make your answer positive if the heat is transferred into the system; make it negative if the heat is transferred into the surroundings.

ANSWER:  
$Q = -230$ kJ

Part I

In a certain process, the energy of the system decreases by 250 kJ. The process involves 480 kJ of work done on the system. Find the amount of heat $Q$ transferred in this process.
Express your answer numerically in kilojoules. Make your answer positive if the heat is transferred into the system; make it negative if the heat is transferred into the surroundings.

ANSWER: \[ Q = -720 \text{ kJ} \]