Summaries of chapters for 1st mid-term
(cannot be brought to the exam)

14 (Oscillations)

GENERAL PRINCIPLES

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

Horizontal spring

\( (F_{\text{net}})_x = -kx \)

Vertical spring

The origin is at the equilibrium position \( \Delta L = mg/k \).

\( (F_{\text{net}})_y = -ky \)

\[ \omega = \sqrt{\frac{k}{m}} \quad T = 2\pi \sqrt{\frac{m}{k}} \]

Pendulum

\( (F_{\text{net}})_t = -\left(\frac{mg}{L}\right)s \)

\[ \omega = \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}} \]

Energy

If there is no friction or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy \( E = K + U \) is conserved.

\[ E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 \]

\[ = \frac{1}{2}m(v_{\text{max}})^2 \]

\[ = \frac{1}{2}kA^2 \]

In a damped system, the energy decays exponentially

\[ E = E_0 e^{-\delta t} \]

where \( \tau \) is the time constant.
14 (Oscillations) (contd.)

**IMPORTANT CONCEPTS**

Simple harmonic motion (SHM) is a sinusoidal oscillation with period $T$ and amplitude $A$.

Frequency $f = \frac{1}{T}$

Angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$

Position $x(t) = A \cos(\omega t + \phi_0)$

$$= A \cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

Velocity $v_x(t) = -v_{max} \sin(\omega t + \phi_0)$ with maximum speed $v_{max} = \omega A$

Acceleration $a_x = -\omega^2 x$

SHM is the projection onto the $x$-axis of uniform circular motion.

$\phi = \omega t + \phi_0$ is the phase

The position at time $t$ is

$$x(t) = A \cos \phi = A \cos(\omega t + \phi_0)$$

The phase constant $\phi_0$ determines the initial conditions:

$$x_0 = A \cos \phi_0 \quad v_{0x} = -\omega A \sin \phi_0$$

**APPLICATIONS**

**Resonance**

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{ext} \approx f_0$ where $f_0$ is the system’s natural oscillation frequency, or resonant frequency.

**Damping**

If there is a drag force $\vec{D} = -b \vec{v}$, where $b$ is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m} \cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = \frac{m}{b}$. 
**15 (Fluids)**

**GENERAL PRINCIPLES**

### Fluid Statics

**Gases**
- Freely moving particles
- Compressible
- Pressure primarily thermal
- Pressure constant in a laboratory-size container

**Liquids**
- Loosely bound particles
- Incompressible
- Pressure primarily gravitational
- Hydrostatic pressure at depth $d$ is $p = p_0 + \rho gd$

### Fluid Dynamics

**Ideal-fluid model**
- Incompressible
- Smooth, laminar flow
- Nonviscous
- Irrotational

**IMPORTANT CONCEPTS**

**Density** $\rho = m/V$, where $m$ is mass and $V$ is volume.

**Pressure** $p = F/A$, where $F$ is the magnitude of the fluid force and $A$ is the area on which the force acts.
- Exists at all points in a fluid
- Pushes equally in all directions
- Constant along a horizontal line
- Gauge pressure $p_g = p - 1$ atm

**Equation of continuity**

$$v_1A_1 = v_2A_2$$

**Bernoulli’s equation**

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli’s equation is a statement of energy conservation.
APPLICATIONS

Buoyancy is the upward force of a fluid on an object.

Archimedes’ principle
The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink \( \rho_{\text{avg}} > \rho_f \) \( F_B < w_o \)
Rise to surface \( \rho_{\text{avg}} < \rho_f \) \( F_B > w_o \)
Neutral buoyant \( \rho_{\text{avg}} = \rho_f \) \( F_B = w_o \)
# 20 (Traveling Waves)

## General Principles

### The Wave Model

This model is based on the idea of a **traveling wave**, which is an organized disturbance traveling at a well-defined **wave speed** $v$.

- In **transverse waves** the particles of the medium move perpendicular to the direction in which the wave travels.
- In **longitudinal waves** the particles of the medium move parallel to the direction in which the wave travels.

A wave transfers **energy**, but no material or substance is transferred outward from the source.

### Important Concepts

The **displacement** $D$ of a wave is a function of both position (where) and time (when).

- A **snapshot graph** shows the wave’s displacement as a function of position at a single instant of time.
- A **history graph** shows the wave’s displacement as a function of time at a single point in space.

A wave traveling in the positive $x$-direction with speed $v$ must be a function of the form $D(x - vt)$.

A wave traveling in the negative $x$-direction with speed $v$ must be a function of the form $D(x + vt)$.

### Sinusoidal Waves

**Sinusoidal waves** are periodic in both time (period $T$) and space (wavelength $\lambda$).

$$D(x, t) = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) + \phi_0 \right]$$

$$= A \sin (kx - \omega t + \phi_0)$$

where $A$ is the **amplitude**, $k = \frac{2\pi}{\lambda}$ is the **wave number**, $\omega = 2\pi f = \frac{2\pi}{T}$ is the **angular frequency**, and $\phi_0$ is the **phase constant** that describes initial conditions.

The fundamental relationship for any sinusoidal wave is $v = \frac{\lambda}{T}$.
Applications

Wave speeds for some specific waves:

- **String** (transverse): \( v = \sqrt{\frac{T_s}{\mu}} \)
- **Sound** (longitudinal): \( v = 343 \text{ m/s in 20°C air} \)
- **Light** (transverse): \( v = \frac{c}{n} \), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum and \( n \) is the material’s index of refraction.

The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency \( f_0 \) emitted.

**Approaching source**

\[
\begin{align*}
  f_+ &= \frac{f_0}{1 - v_s/v} \\
  f_+ &= (1 + v_s/v)f_0
\end{align*}
\]

**Receding source**

\[
\begin{align*}
  f_- &= \frac{f_0}{1 + v_s/v} \\
  f_- &= (1 - v_s/v)f_0
\end{align*}
\]

The wave **intensity** is the power-to-area ratio

\[
I = \frac{P}{A}
\]

For a circular or spherical wave

\[
I = \frac{P_{\text{source}}}{4\pi r^2}
\]

The Doppler effect for light uses a result derived from the theory of relativity.
**GENERAL PRINCIPLES**

**Principle of Superposition**
The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.

**IMPORTANT CONCEPTS**

**Standing waves** are due to the superposition of two traveling waves moving in opposite directions.

Standing wave pattern:
- Antinodes
- Nodes
- Node spacing is $\frac{1}{2}\lambda$

The amplitude at position $x$ is

$$A(x) = 2a \sin kx$$

where $a$ is the amplitude of each wave.

The boundary conditions determine which standing wave frequencies and wavelengths are allowed.

**Interference**

In general, the superposition of two or more waves into a single wave is called interference.

**Maximum constructive interference** occurs where crests are aligned with crests and troughs with troughs. These waves are in phase. The maximum displacement is $A = 2a$.

**Perfect destructive interference** occurs where crests are aligned with troughs. These waves are out of phase. The amplitude is $A = 0$.

Interference depends on the phase difference $\Delta \phi$ between the two waves.

Constructive:

$$\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2m\pi$$

Destructive:

$$\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2(m + \frac{1}{2})\pi$$

$\Delta r$ is the path-length difference of the two waves and $\Delta \phi_0$ is any phase difference between the sources. For identical sources (in phase, $\Delta \phi_0 = 0$):

- Interference is constructive if the path-length difference $\Delta r = m\lambda$.
- Interference is destructive if the path-length difference $\Delta r = (m + \frac{1}{2})\lambda$.

The amplitude at a point where the phase difference is $\Delta \phi$ is

$$A = 2a \cos \left( \frac{\Delta \phi}{2} \right)$$
APPLICATIONS

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends.

\[ \lambda_m = \frac{2L}{m} \quad f_m = m \frac{v}{2L} = mf_1 \]

where \( m = 1, 2, 3, \ldots \)

The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

\[ \lambda_m = \frac{4L}{m} \quad f_m = m \frac{v}{4L} = mf_1 \]

where \( m = 1, 3, 5, 7, \ldots \)

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.

The beat frequency between waves of frequencies \( f_1 \) and \( f_2 \) is

\[ f_{\text{beat}} = f_1 - f_2 \]