Lecture 8

- standing waves on string and tubes
- Interference in 1D: superposition of waves in same direction
  graphical and mathematical phase and path-length difference
  application to thin films
- Interference in 2/3D
Transverse standing waves
• generate standing waves on string fixed at both ends: traveling wave encounters a boundary...

(a) Before: String with smaller wave speed
After: String with larger wave speed

Discontinuity where the wave speed increases

(b) Before: String with larger wave speed
After: The reflected pulse is inverted.

Discontinuity where the wave speed decreases

(c) Before: Boundary
After: The reflected pulse is inverted and its amplitude is unchanged.

(phase change of $\pi$ on reflection)
Standing waves on string

- Reflection does not change f, \( \lambda \)

- boundary condition (constraint obeyed at edge) \( D = 0 \) at ends (nodes) at all times

\[
A(x) = 2a \sin kx = 0 \text{ at } x = 0 \text{ and at } x = L \\
\text{if } \sin kL = 0 \Rightarrow kL = m\pi
\]

- allowed standing waves (traveling for other...)

\[
\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \ldots
\]

\[
f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = \frac{m}{2L} \quad m = 1, 2, 3, 4, \ldots
\]

- normal modes:

  **fundamental** (smallest) frequency:

\[
f_1 = \frac{v}{2L}, \quad \lambda = 2L \text{ (only half } \lambda \text{ and no node in-between)}
\]

**harmonics:** \( f_m = mf_1 \ (m = 2, 3, \ldots) \) \( (m - 1 \text{ nodes and } m \text{ antinodes in-between}) \)

master formulae
Example

- A 121-cm-long, 4.0 g string oscillates in its m=3 mode with a frequency of 180 Hz and a maximum amplitude of 5.0 mm. What are (a) the wavelength and (b) the tension in the string?
Standing EM waves

• light wave has node at each mirror...similar to string....

• microwave oven: $\lambda \sim \text{cm}$ vs. size of oven $\sim 10$’s cm $\Rightarrow$ turntable to avoid part of food being always a node
Standing Sound Waves (I)

- closed end: node (air can’t oscillate)
- **open** end (sound wave partly reflected back into tube): **antinode**

(a) Closed-closed

(b) Open-open

\[ \lambda_m = \frac{2L}{m} \]
\[ f_m = m \frac{v}{2L} = mf_1 \]

\( m = 1, 2, 3, 4, \ldots \)

(...like string)
Standing Sound Waves (II)

- fundamental mode: $1/4 \lambda$ in length $L$ (cf. $1/2$ for closed-closed or open-open) → frequency is half of open-open/closed-closed...

\[
\begin{align*}
\lambda_m &= \frac{4L}{m} \\
\left\{ \begin{array}{l}
f_m = m \frac{v}{4L} = mf_1 \\
m = 1, 3, 5, 7, \ldots \\
\text{(open-closed tube)}
\end{array} \right.
\]

sound speed in air

Musical Instruments

- **stringed**: fundamental frequency: $f_1 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{T_s}{\mu}}$
  - speed along string

- **wind**: change effective length by covering holes/opening valves
Interference: basic set-up

- standing waves: superposition of waves traveling in **opposite** direction (**not** a traveling wave) is **one** type of interference

- 2 waves traveling in **same** direction:

\[ D_1(x_1, t) = a \sin (kx_1 - \omega t + \phi_{10}) = a \sin \phi_1 \]
\[ D_2(x_2, t) = a \sin (kx_2 - \omega t + \phi_{20}) = a \sin \phi_2 \]

- Phase constants tell us what the source is doing at \( t = 0 \)
Interference: graphical

(a) Constructive interference
These two waves are in phase. Their crests are aligned.

D_1(x, t) = D_2(x, t) \Rightarrow \phi_1 = \phi_2 \pm 2\pi m
D_{net} = 2D_1 \text{ (or } D_2): A = 2a

Their superposition produces a wave with amplitude 2a. This is constructive interference.

(b) Destructive interference
These two waves are out of phase. The crests of one wave are aligned with the troughs of the other.

D_1(x, t) = -D_2(x, t) \Rightarrow D_{net}(x) = 0

crest of wave 1 aligned with trough of wave 2: out of phase

aligned crest-to-crest and trough-to trough: in-phase
Phase and path-length difference (t drops out)

- **net** phase difference determines interference: 2 contributions
  
  (i) **path-length difference:**
  \[
  \Delta x = x_2 - x_1 \quad \text{(distance between sources: extra distance traveled by wave 2; independent of observer)}
  \]

  (ii) **inherent phase difference:**
  \[
  \Delta \phi_0 = \phi_{10} - \phi_{20}
  \]

  - Perfect destructive interference:
    \[
    \Delta \phi = 2\pi \frac{\Delta x}{\lambda} + \Delta \phi_0 = 2\left(m + \frac{1}{2}\right)\pi \text{ rad} \quad m = 0, 1, 2, 3, \ldots
    \]

  - Maximum constructive interference:
    \[
    \Delta \phi = 2\pi \frac{\Delta x}{\lambda} + \Delta \phi_0 = 2m\pi \text{ rad} \quad m = 0, 1, 2, 3, \ldots
    \]

  (a) The sources are out of phase.
  
  (b) Identical sources are separated by half a wavelength.
Example

- Two in-phase loudspeakers separated by distance d emit 170 Hz sound waves along the x-axis. As you walk along the axis, away from the speakers, you don’t hear anything even thought both speakers are on. What are three possible values for d? Assume a sound speed of 340 m/s.

(\Delta x \Rightarrow \Delta \phi \text{ independent of observer, depends on distance between sources in 1D cf. 2/3D})
Identical sources: $\Delta \phi_0 = 0$

maximum constructive interference: $\Delta x = m\lambda$
perfect destructive interference: $\Delta x = (m + 1/2)\lambda$

Interference: mathematical

$$D = D_1 + D_2 = A \sin (kx_{avg.} - \omega t + \phi_{0 \ avg.})$$
with
$$x_{avg.} = (x_1 + x_2)/2; \phi_{0 \ avg.} = (\phi_{10} + \phi_{20})/2$$
and
$$A = 2a \cos \frac{\Delta \phi}{2} \quad (\Rightarrow \text{independent of } x \text{ of observer: see e.g.})$$

⇒ traveling wave (due to $x_{avg.} - vt$) with
$$x_{avg.} = x \text{ of observer} + x \text{ of midpoint of sources}$$

$A$ varies from
0 for $\Delta \phi = (2m + 1)\pi$ (perfect destructive interference) to
2$a$ for $\Delta \phi = 2m\pi$ (maximum constructive interference)

(confirm graphical...)

• In general, neither exactly out of nor in phase

For $\Delta \phi = 40^\circ$, the interference is constructive but not maximum constructive.

For $\Delta \phi = 160^\circ$, the interference is destructive but not perfect destructive.
Application to thin films

- reflection from boundary at which index of refraction increases (speed of light decreases) → phase shift of \( \pi \)

- 2 reflected waves interfere: from air-film and film-glass boundaries (\( n_{air} = 1 < n_{film} < n_{glass} \))

\[ \Delta \phi = 2\pi \frac{\Delta x}{\lambda_f} - \Delta \phi_0 \]

- wave 2 travels twice thru' film

\[ \lambda_f = \frac{\lambda}{n} \]

\[ \Rightarrow \text{constructive for } \lambda_C = \frac{2nd}{m}, \ m = 1, 2, \ldots \]

\[ \Rightarrow \text{destructive for } \lambda_D = \frac{2nd}{m-\frac{3}{2}}, \ m = 1, 2, \ldots \] (anti-reflection coating for lens)
Mnemonic for phase shifts at boundaries

• “Low to high, phase shift of $\pi$ ”
• “High to low, phase shift no”

• Low and high above refers to index of refraction
  high and low speeds of light
• works for strings as well...
Interference in 2/3 D

- Similar to 1D with \( x \rightarrow r \): 
  \[ D(r, t) = a \sin(kr - \omega t + \phi_0) \]

after \( T/2 \), crests ↔ troughs...points of constructive/destructive interference same

Maximum constructive interference:
\[ \Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2m\pi \]

Perfect destructive interference:
\[ \Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2\left(m + \frac{1}{2}\right)\pi \]

\( m = 0, 1, 2, \ldots \)

\( \Delta r \) depends on observer

measure \( r \) by counting rings...

constant a approx.

Points of constructive interference.
A crest is aligned with a crest, or a trough with a trough.

Points of destructive interference.
A crest is aligned with a trough of another wave.

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Example

- Two out-of-phase radio antennas at $x = +/- 300$ m on the x-axis are emitting 3.0 MHz radio waves. Is the point $(x, y) = (300 \text{ m}, 800 \text{ m})$ a point of maximum constructive interference, perfect destructive interference, or something in between?
Interference in 1 vs. 2/3 D

\[ \Delta r \ (\Rightarrow \ \Delta \phi) \text{ depends on observer in } 2/3 \text{ (vs. not in 1D)} \]

In 2/3D, \( D_{net} = D_1 + D_2 = A \sin (kr_{avg} - \omega t) \) with \( A = 2a \cos (\Delta \phi/2) \)...as in 1D...

...but \( A \) (due to \( \Delta \phi \)) depends on observer in 2/3D...

In general, not a traveling wave (cf. 1D)
Picturing interference in 2/3 D

- Antinodal and nodal lines: line of points with same $\Delta r$, A → traveling wave along it

**Strategy for interference problems**

- assume circular waves of same amplitude
- draw picture with sources and observer at distances $r_1, 2$
- note $\Delta \phi_0$

Maximum constructive interference:

$$\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2m\pi$$

Perfect destructive interference:

$$\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2\left(m + \frac{1}{2}\right)\pi$$

$m = 0, 1, 2, \ldots$

$(\Delta r \to \Delta x \text{ for 1D})$