Lecture 23

- Conductor in equilibrium
- Capacitance
- Combinations of capacitors (series and parallel)
- Energy stored in capacitor (electric field)
Conductor in Equilibrium

• $E = 0$ inside conductor in equilibrium (otherwise, charges would move)

• Entire conductor at same $V$, surface is equipotential $\rightarrow \vec{E} \perp$ to surface
**Capacitance**

(a) $\Delta V_c = 0$  
(b) $\Delta V_{wire} = 0$

$V = Ed; \quad E = \frac{Q}{\varepsilon_0 A}$;  
$C \equiv \frac{\varepsilon_0 A}{d} \Rightarrow$

$q = C \Delta V_c$ \hspace{1em} (charge on a capacitor)

Units of $C$:  
1 farad = 1 F $\equiv$ 1 C/V

$C$ geometric property  
(of any two electrodes)

**Combinations...**

The potential differences along the wires create a current that moves charge from one capacitor plate to the other.

When $\Delta V_c = \Delta V_{bat}$, the current stops and the capacitor is fully charged.

The circuit symbol for a capacitor is two parallel lines.

Series capacitors are joined end to end in a row.

Parallel capacitors are joined top to top and bottom to bottom.
Capacitors in Parallel (a)  
- same $\Delta V_C \Rightarrow$

$$C_{eq} = \frac{\Delta Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C}$$

$$= \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Capacitors in Series
- same charge $Q$  

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q}$$

$$= \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q}$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$
Circuit analysis

• combine elements into single equivalent; reverse process to calculate for each element

\[ \begin{align*}
C_1 &= 3 \, \mu F \\
C_2 &= 5 \, \mu F \\
C_3 &= 1 \, \mu F
\end{align*} \]
Energy Stored in Capacitor (Electric Field)

- Potential energy of dq + capacitor increases by \( dU = dq \Delta V = \frac{q dq}{C} \)

- Total energy transferred from battery to capacitor:

\[
U_C = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} = \frac{1}{2} C (\Delta V_C)^2
\]

- Like spring \((1/2k (\Delta x)^2)\): discharged/released, potential to kinetic...

- Energy stored in E (real!): using \( \Delta V_C = Ed \) and \( C = \epsilon_0 A/d \), \( U_C = \frac{\epsilon_0}{2} (Ad) E^2 \)

\[
\epsilon_E = \frac{\text{energy stored}}{\text{volume stored in}} = \frac{U_c}{Ad} = \frac{\epsilon_0}{2} E^2
\]

The instantaneous charge on the plates is \( \pm q \).

The charge escalator does work \( dq \Delta V \) to move charge \( dq \) from the negative plate to the positive plate.

The capacitor’s energy is stored in the electric field in volume \( Ad \) between the plates.
Dielectrics I

- vacuum filled charged; battery disconnected; fill with dielectric:

\[ \Delta V_C < (\Delta V_C)_0 \Rightarrow C = \frac{Q}{\Delta V_C} > \frac{Q_0}{(\Delta V_C)_0} = C_0 \]

- 2 steps: polarization (dipoles align separation of charge \( \rightarrow \) \( E \) induced)

(a) The insulator is polarized.

(b) The polarized insulator—a dielectric—can be represented as two sheets of surface charge.
Dielectrics II: superposition of $E$

If battery kept connected: fixed, more charge flows:

\[ \bar{E} = \bar{E}_0 + \bar{E}_{\text{induced}} = (E_0 - E_{\text{induced}}, \text{ from positive to negative}) \]

Dielectric constant: \( \kappa \equiv \frac{E_0}{E} \geq 1; \kappa_{\text{vacuum}} = 1 \)

\[ \Delta V_C = Ed = \frac{E_0}{\kappa} d = \frac{(\Delta V_C)_0}{\kappa} \Rightarrow \]

\[ C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/\kappa} = \kappa \frac{Q_0}{(\Delta V_C)_0} = \kappa C_0 \text{ (increases!)} \]

- if battery kept connected: \( \Delta V_C \) fixed, more charge flows: \( Q = \kappa Q_0 \)