Lecture 37

- Kirchhoff’s Laws and Basic Circuit
- Energy and Power
- Resistors in Series
Kirchhoff’s Laws

- circuit analysis: finding potential difference across and current in each component

- junction law (charge conservation)
  \[ \sum I_{in} = \sum I_{out} \]

- loop law (energy conservation)
  \[ \Delta V_{loop} = \sum_i (\Delta V)_i = 0 \]

- strategy:
  assign current direction
  travel around loop in direction of current
  \[ V_{bat} = \pm \mathcal{E}; \quad V_R = -IR \]
  apply loop law

\[ \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0 \]
Basic Circuit

• junction law not needed

• ideal wires: no potential difference

• loop law: \( \Delta V_{loop} = \Delta V_{bat} + \Delta V_R = 0 \)
  \[ \Delta V_{bat} = +\mathcal{E}; \]
  \[ \Delta V_R = V_{\text{downstream}} - V_{\text{upstream}} = -IR \Rightarrow \mathcal{E} - IR = 0; \]
  \[ I = \frac{\mathcal{E}}{R}; \Delta V_R = -IR = -\mathcal{E} \]

1. Draw circuit diagram.

2. The orientation of the battery indicates a clockwise current, so assign a clockwise direction to \( I \).

3. Determine \( \Delta V \) for each circuit element.
A more complex circuit

- charge can flow “backwards” thru’ battery: choose cw direction for current (if solution negative, current is really ccw)

- loop law: \( \Delta V_{bat_1} + \Delta V_{R_1} + \Delta V_{bat_2} + \Delta V_{R_2} = 0 \)

\[
\begin{align*}
\Delta V_{bat_1} &= +\mathcal{E}_1; \\
\Delta V_{bat_2} &= -\mathcal{E}_2; \\
\Delta V_{R_1} &= -IR_1; \\
\Delta V_{R_1} &= -IR_2 \\
\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 &= 0 \\
I &= \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6 \, \text{V} - 9 \, \text{V}}{4 \Omega + 2 \Omega} = -0.5 \, \text{A}...
\end{align*}
\]

(expected: 9 V battery “dictates” direction...)

\[
\Delta V_{R_1} = -IR_1 = +2.0 \, \text{V}...
\]
Energy and Power

- In battery: chemical energy potential...of charges: \( \Delta U = q \Delta V_{bat} = q \mathcal{E} \)

- Power (rate at which energy supplied to charges): \( P_{bat} = \frac{dU}{dt} = \frac{dq}{dt} \mathcal{E} \)

\[ P_{bat} = I \mathcal{E} \quad \text{(power delivered by an emf)} \]

- In resistor: work done on charges \( qEd \rightarrow \) kinetic (accelerate) between collisions \( \rightarrow \) thermal energy of lattice after collisions

After many collisions over length \( L \) of resistor:
\[ \Delta E_{th} = qEL = q\Delta V_R \]

Rate at which energy is transferred from current to resistor:
\[ P_R = \frac{dE_{th}}{dt} = \frac{dq}{dt} \Delta V_R = I \Delta V_R \]

For basic circuit: \( P_R = P_{bat} \) (energy conservation)

Using Ohm’s law:
\[ P_R = I \Delta V_R = I R = \frac{(\Delta V_R)^2}{R} \quad \text{(power dissipated by a resistor)} \]

\( E_{chem} \rightarrow U \rightarrow K \rightarrow E_{th} \rightarrow \text{light...} \)

- Common units: \( P_R \) kW in \( \Delta t \) hours \( \rightarrow P_R \Delta t \) kilowatt hours
Resistors in Series

(a) Two resistors in series

\[
\Delta V_1 = IR_1; \quad \Delta V_2 = IR_2 \Rightarrow \Delta V_{ab} = I (R_1 + R_2)
\]

(b) An equivalent resistor

\[
R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2
\]

- current same in each resistor: \(\Delta V_1 = IR_1; \Delta V_2 = IR_2 \Rightarrow \Delta V_{ab} = I (R_1 + R_2)\)
- equivalent resistor: \(R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2\)

\[
R_{eq} = R_1 + R_2 + \cdots + R_N \quad \text{(series resistors)}
\]

Ammeters

- measures current in circuit element (placed in series):

\[
R_{eq} = R_{load} + R_{ammeter}
\]

ideal, \(R_{ammeter} = 0 \Rightarrow \text{current not changed}\)