Lecture 36

• Capacitance
• Combinations of capacitors (series and parallel)
• Energy stored in capacitor (electric field)
• start chapter 31 (Fundamentals of Circuits)
Capacitance

$\Delta V_c = 0 \rightarrow \Delta V_{wire} = 0$

$V = Ed; \ E = \frac{Q}{\epsilon_0 A}$

$C \equiv \frac{Q}{\epsilon_0 A} \Rightarrow$

$Q = C \Delta V_c$  \ (charge on a capacitor)

Units of C:

1 farad = 1 F = 1 C/V

$C$ geometric property

(of any two electrodes)

Combinations...

The potential differences along the wires create a current that moves charge from one capacitor plate to the other.

When $\Delta V_c = \Delta V_{bat}$, the current stops and the capacitor is fully charged.

The circuit symbol for a capacitor is two parallel lines.

Parallel capacitors are joined top to top and bottom to bottom.

Series capacitors are joined end to end in a row.
### Capacitors in Parallel

- **same $\Delta V_c$**

$$C_{eq} = \frac{\Delta Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_1}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

- Parallel capacitors have the same $\Delta V_c$.

### Capacitors in Series

- **same charge $Q$**

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q}$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

- Series capacitors have the same $Q$.

- **Same total charge as $C_1$ and $C_2$**

$$Q = \Delta V_1 + \Delta V_2$$

- No net charge on this isolated segment

- **Same total potential difference as $C_1$ and $C_2$**

$$\Delta V_C = \Delta V_1 + \Delta V_2$$
Circuit analysis

- combine elements into single equivalent; reverse process to calculate for each element
Energy Stored in Capacitor (Electric Field)

- Potential energy of \( dq \) + capacitor increases by \( dU = dq \Delta V = \frac{dq q}{C} \)

- Total energy transferred from battery to capacitor:
  \[
  U_C = \frac{1}{C} \int_0^Q dq d = \frac{Q^2}{2C} = \frac{1}{2} C (\Delta V_C)^2
  \]

- Like spring \((1/2k(\Delta x)^2)\): discharged/released, potential to kinetic...

- Energy stored in \( E \) (real!):
  Using \( \Delta V_C = Ed \) and \( C = \epsilon_0 A/d \), \( U_C = \frac{\epsilon_0}{2} (Ad) E^2 \)

\[
\epsilon_E = \frac{\text{energy stored}}{\text{volume stored in}} = \frac{U_c}{Ad} = \frac{\epsilon_0}{2} E^2
\]

The instantaneous charge on the plates is \( \pm q \). The charge escalator does work \( dq \Delta V \) to move charge \( dq \) from the negative plate to the positive plate.

The capacitor’s energy is stored in the electric field in volume \( Ad \) between the plates.

Capacitor plate with area \( A \)
Chapter 31 (Fundamentals of Circuits)

• understand fundamental principles of electric circuits; direct current (DC): battery’s potential difference, currents constant

Resistors and Ohm’s law

\[ I = \frac{\Delta V}{R} \]

(cause and effect)

• resistors: circuit elements with resistance larger than wires used to limit current

1. A battery is a source of potential difference.

\[ \Delta V_{\text{wire}} = \Delta V_{\text{bat}} \]

2. The wire is a conductor. Charges can move through the wire as a current.

3. The potential difference creates an electric field \( E = \Delta V_{\text{wire}} / L \) inside the wire.

4. The electric field causes current density \( J = \sigma E \) and current \( I = \Delta V_{\text{wire}} / R \) in the wire. The current is in the direction of decreasing potential.
Ohmic and Nonohmic materials; Ideal Wire Model

- **ideal wires:** \( R = 0 \Rightarrow \Delta V = 0 \) even \( I \neq 0 \)

- **resistors:**
  
  10 to \(10^6\) \(\Omega\)

- **ideal insulators:**
  
  \( R = \infty \Rightarrow I = 0 \) even if \( \Delta V \neq 0 \)
Circuit Elements and Diagrams

• circuit diagram: logical picture of connections (replace pictures of circuit elements by symbols)