Midterm 3

You must show your work to get full credit.

1. Jack (who has mass $m$) stands on a plank of mass $M$ and length $L$ that is balanced at the middle at an angle $\theta$, with his end touching the ground. His twin brother Mack (who has the same mass) lands on the other end of the plank with speed $v$. Find the speed of Mack just before his end of the plank touches the ground. Assume that both Mack and Jack stay on the plank during this motion. The moment of inertia of the plank about its center is $\frac{1}{12}ML^2$.

![Diagram of Jack and Mack on a plank]

2. A canon fires a cannonball with speed $\frac{2}{3}c$ at an angle of $30^\circ$ above the horizontal, as seen by an observer at rest with respect to the earth. What is the speed of the cannonball as seen by an observer running horizontally with speed $\frac{1}{2}c$ toward the canon? Neglect gravity. *Hint:* Write an equation for the horizontal and vertical position of the cannonball in the ground frame (with constant speed since we are neglecting gravity). Then use the Lorentz transformation equations to transform it into the reference frame of the runner.

![Diagram of cannonball being fired]

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\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}.
\]
3. Find integrals for the horizontal and vertical components of the gravitational force on a point mass $m$ due to a rod of length $L$ and mass $M$ positioned as shown below. Do this directly from Newton’s law of gravity, not by differentiating the gravitational potential. Your answer should be a definite integral expressed entirely in terms of $G$, $m$, $M$, $L$, $x$, and an integration variable. *You do not have to do the integrals!*