(a) The normal force between mass 1 and the inclined plane $N = m_1 g \cos \theta$ will generate friction force between $\pm N \mu_s = m_1 m_2 g \cos \theta$.

For mass 2

\[ T - m_2 g = 0 \quad \Rightarrow \quad T = m_2 g \]

For mass 1 on the direction of inclined plane

\[ T - m_1 g \sin \theta \pm N \mu_s = 0 \]

\[ \Rightarrow m_1 g - m_2 g \sin \theta \pm \mu_s m_2 g \cos \theta = 0 \]

\[ \Rightarrow m_1 = \frac{m_2}{\sin \theta \pm \mu_s \cos \theta} \]

(b) If $m_1$ is large, $m_1$ will move downward with friction force $N \mu_s$ upward ("+" direction).

For mass 2

\[ T - m_2 g = (\pm)m_2 a \quad \Rightarrow \quad T = m_2 (g - a) \]

For mass 1

\[ T - m_1 g \sin \theta + \mu_s N = m_1 a \]
\[ m_2 (g - a) - m_1 g \sin \theta + \mu_2 m_1 g \cos \theta = m_1 a \]

\[
\therefore a = \frac{m_2 - m_1 \sin \theta + \mu_2 m_1 \cos \theta}{m_1 + m_2} \cdot g
\]
The normal force between the package exerts on the cart is

\[ N = Mg + F \text{ain} \theta \]

which will generate a friction force

\[ N \mu_s = \mu_s (Mg + F \text{ain} \theta) \]

If we assume that the package and the cart have different accelerations, \( a_m \neq a_m \).

For package

\[ a_m \rightarrow N = (Mg + F \text{ain} \theta) \]

\[ F \text{wo} - \mu (Mg + F \text{ain} \theta) = MA_m \]

For the cart

\[ a_m \rightarrow N = (Mg + F \text{ain} \theta) \]

\[ \downarrow mg \]
\[(Mg + F\sin\theta) a = m A_m\]

(a) Now if the package does not slide, \(A_m = A_M = 0\)

\[
\begin{align*}
F \cos\theta - u_s(Mg + F\sin\theta) &= MA_M \\
u_s(Mg + F\sin\theta) &= MA_M
\end{align*}
\]

\[
\Rightarrow F \cos\theta - u_s(Mg + F\sin\theta) = M \frac{u_s(Mg + F\sin\theta)}{m}
\]

\[
F = \frac{u_sMg (1 + \frac{M}{m})}{\cos\theta - u_s\sin\theta (1 + \frac{M}{m})}
\]

(b) If the force is greater than that above, then \(A_M = A_m\)

\[
A_M = \frac{F \cos\theta - m u_s(Mg + F\sin\theta)}{M}
\]

\[
A_m = \frac{m u_s(Mg + F\sin\theta)}{m}
\]
For \( m_1 \),
\[
T_1 - T_2 = m_1 \left( \frac{2\pi}{I} \right)^2 L_1
\]

For \( m_2 \),
\[
T_2 = m_2 \left( \frac{2\pi}{I} \right)^2 (L_1 + L_2)
\]

\[
\Rightarrow T_1 = T_2 + m_1 \left( \frac{2\pi}{I} \right)^2 L_1
\]
\[
= \left( \frac{2\pi}{I} \right)^2 \left( m_1 L_1 + m_2 L_1 + m_2 L_2 \right) > T_2
\]

\[
T_1 > T_2
\]

Because \( m_1 \) must have a net inward force to keep rotation, \( T_1 \) must be greater than \( T_2 \).
To travel with constant speed $v$ on circular section the car need a inward centripetal acceleration $m \frac{v^2}{R}$ which can only be provided by gravity

$$mg \sin \left( \frac{\pi}{2} - \theta + \phi \right) - N = m \frac{v^2}{R}$$

To keep the car on the hill, $N$ must be greater than zero

$$mg \sin \left( \frac{\pi}{2} - \theta + \phi \right) > m \frac{v^2}{R}$$

$$v < \sqrt{g \sin \left( \frac{\pi}{2} - \theta + \phi \right)}$$

The maximum possible speed is when $\phi = 0$

$$v = \sqrt{g \cos \theta}$$

If $v$ is slightly greater than this value, the car will fly into the air immediately after it go into the circular section.
5.

(a) The radius of the earth is \(6.39 \times 10^6 \text{m}\)

The angular velocity is \(\omega = \frac{2\pi}{T}\)

\[= \frac{2\pi}{24 \text{ hr} \times 60 \text{ min} \times 60 \text{ sec}}\]

\[= 7.27 \times 10^{-5} \text{ rad/sec}\]

The centripetal acceleration \(a = \omega^2 r\)

But \(v = \omega \times \text{coo}\)

\[\Rightarrow a = \omega^2 r = \omega^2 R \times \text{c}\]

\[= (7.27 \times 10^{-5})^2 \times (6.39 \times 10^6) \times \text{c}\]

\[= 0.0337 \times \text{c}\]

\[= 3.37 \times \text{c}\]

Toward the axis of earth rotation

not the center of the earth
(b) For an object in the earth's rotation frame, it will experience a gravity force toward the center of the earth and an outward force caused by rotation.

Therefore, the apparent weight will be slightly lighter than that caused by gravity only.

(c) at the equator:

\[ g \rightarrow 3.37 \text{ m/s}^2 \quad \Rightarrow \quad g = 981.4 \text{ cm/sec}^2 \]

at 45°

\[ (g - 2.38)^2 + 2.38^2 = 981^2 \quad \Rightarrow \quad g = 983.4 \text{ cm/sec}^2 \]

Because the earth is not a perfect sphere, its gravitational acceleration is different at different latitude.
6. (a) No, there is no downward force. The force working on the astronaut is the force exerted by the floor of the station which is upward to the center of rotation to provide the required centripetal force.

An object in an accelerating frame will experience an acceleration opposite to that of the frame. In this case, the astronaut will feel a downward force because the rotating frame has an upward acceleration.

\[ m \cdot \omega^2 R = m \ddot{y}g \]

\[ \Rightarrow \left( \frac{2\pi}{T} \right)^2 R = g \]

\[ T = 2\pi \sqrt{\frac{R}{g}} \]

(b) After the astronaut release the ball from height \( h \), it will move at tangential direction with velocity

\[ v = \omega (R-h) = \frac{2\pi}{T} (R-h) \]

After time \( \Delta t \), the ball hits the floor at point \( B \), and the astronaut moves to point \( C \).
The rotation angular velocity \( \omega = \frac{2\pi}{T} \)

After release, the velocity of the ball is
\[
\mathbf{v} = \omega (\mathbf{r} - \mathbf{h}) = \omega \mathbf{r} \cos \theta
\]

When hitting the floor after \( \Delta t \), the ball has traved
\[
\mathbf{v} \Delta t = \omega R \cos \theta \Delta t = R \sin \theta
\]

\[ \Rightarrow \Delta t = \frac{1}{\omega \cos \theta} \frac{\sin \theta}{\omega} = \frac{1}{\omega \tan \theta} \]

In the period, the astronaut rotates to \( C \). The arch between \( A \) \( C \) is

\[
\phi \cdot R = (\omega \cdot \Delta t) R = R \tan \theta
\]

The arch between \( AB \) is
\[
\text{OR}
\]
where \( \theta = \cos^{-1} \frac{R-h}{R} \)}

The arch between \( BC \) is
\[
R (\tan \theta - 0) > 0
\]
Since \( R(\tan \theta - \theta) > 0 \), the ball will not fall straight down but fall behind the astronaut.