The effects of relativity can be summarized in the Lorentz transformation equations, which relate the coordinates of events in different reference frames. Consider two frames of reference, where the primed frame is moving relative to the unprimed frame with speed $v$, as shown below.

If the origins of the two reference frames coincide at $t = 0$ measured in both frames, then we expect that the formula relating the coordinates in the two frames are

$$x' = x - vt,$$
$$y' = y,$$
$$z' = z,$$
$$t' = t.$$

These are the Galilean transformations. Einstein’s theory replaces these with the so-called Lorentz transformations

$$x' = \gamma(x - vt),$$
$$y' = y,$$
$$z' = z,$$
$$t' = \gamma(t - vx/c^2),$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$
Note that the Lorentz transformations reduce to the Galilean transformations for \( v \ll c \). In this homework you will use these transformation to reproduce various features of special relativity.

1. Consider a rod of length \( L_0 \) at rest in the unprimed frame. The left end is at \( x = 0 \) and the right end is at \( x = L_0 \).

(a) Show that according to the Galilean transformations, the coordinates of the left end of the rod are

\[ x'_{\text{left}} = -vt' \]

and the coordinates of the right end are

\[ x'_{\text{right}} = L_0 - vt'. \]

From this, we can see that the length of the rod measured in the moving frame is

\[ x'_{\text{right}} - x'_{\text{left}} = L_0. \]

(b) Using the Lorentz transformations, find the coordinates of the left and right end of the rod. Check that your results reduce to those of part (a) when \( v \ll c \). Find the length of the rod in the moving frame. Does your result agree with the Lorentz contraction discussed in the book?

2. A clock at the origin of the unprimed coordinate system ticks once at \( t = 0 \) and then again at \( t = T_0 \).

(a) Show that according to the Galilean transformations, the first tick takes place at

\[ x'_1 = 0, \quad t'_1 = 0, \]

and the second tick takes place at

\[ x'_2 = -vT, \quad t'_2 = T. \]

From this, we see that the length of a tick as measured in the primed frame is

\[ t'_2 - t'_1 = T. \]

(b) Using the Lorentz transformations, find the space and time coordinates of the first and second tick in the primed reference frame. Check that your results reduce to those of part (a) when \( v \ll c \). Find the time for the clock to tick measured in the primed frame. Does your result agree with the time dilation discussed in the book?
3. A particle is moving with constant speed \( v_0 \) in the unprimed coordinate system:

\[
x_p = v_0 t.
\]

(a) Show that according to the Galilean transformations the particle’s position in the primed coordinates is

\[
x_p' = (v_0 - v)t'.
\]

From this we see that the speed of the particle as measured in the primed frame is \( v_0 - v \).

(b) Using the Lorentz transformations, find the particle’s position in the primed coordinates. Use this to find the velocity of the particle in the primed reference frame. Check that this reduces to the formula in part (a) when \( v \ll c \). Verify that if \( v_0 = c \), the particle moves with speed \( c \) in the primed reference frame.

Study Problems

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