Notes on problem 10.23

The parallel axis theorem can be used for calculating the contributions of all three rods to the total $I$.

For the rod parallel to $z$, all of the mass is located at the same, or nearly the same distance, $L/2$, from the rotation axis. $I_{CM}$ for rotation around the long axis is $I_{CM} = \frac{1}{2}Mr^2$, therefore, $I = I_{CM} + M\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}ML^2$ from the parallel axis theorem. (the $\cong$ symbol because $r << L$).

For a rod rotating about an axis perpendicular to its long axis, $I_{CM} = \frac{1}{12}ML^2$

Therefore for the $x$ and the $y$ rods, $I = I_{Cm} + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$ from the parallel axis theorem.

The total $I$ is the sum of these three contributions: $I_{total} \cong \frac{1}{12}ML^2$. The error is just the $I_{CM} = \frac{1}{2}Mr^2$ from the $z$ rod. And for $r << L$ this error is negligible.