This was the question:

a) If I take a mass of 0.25 kg, swing it around in a circle of radius 1 meter at a speed of 5 m/s, what is its kinetic energy?

b) If I then pull in the string, so that it’s turning in a radius of 1/2 meter, how much work did I do in pulling it in?

a) The initial kinetic energy is just \[ \frac{1}{2}mv^2 = \frac{1}{2} (0.25 \text{ kg}) (5 \text{ m/s})^2 = 3 \text{ Joules}. \]

But we could also find that another way: \[ \frac{1}{2}I\omega^2 \]

So we need to know I and \( \omega \).

I = MR^2 = (0.25 \text{ kg}) (1 \text{ m})^2 = 0.25 \text{ kg-m}^2

\( \omega \). Well, there are lots of ways to find \( \omega \), and any one at a time will work! One way is to just say \( v = r\omega \), but if we do it that way, well, that’s just re-doing the derivation of \( \frac{1}{2}I\omega^2 \)...and where’s the fun in that?

So, I thought to find it another way: Find the time it takes to go once around the circle, T, and then \( \omega \) is just \( 2\pi/T \), since once around the circle is \( 2\pi \) radians.

So – how long does it take to go around the circle? The distance around is \( 2\pi r = 2\pi \) meters, or about 6 meters, and it’s moving at a speed of 5 m/s, so it takes \( 6/5 \) seconds to go around, or 1.2 seconds. So, yes, \( \omega = 2\pi/1.2 \text{ sec} = 5 \text{ radians/second} \).

And that brings us back to the initial kinetic energy is \[ \frac{1}{2}I\omega^2 = \frac{1}{2} (0.25 \text{ kg-m}^2) (5 \text{ rad/s})^2 = 3 \text{ Joules}. \]

b) From there, I pull in the cord to 1/2 meter, and the question is how much work did I do?

Angular momentum is conserved, because when I pull it in toward the center, all the force is radial and if all the force is radial, there’s no torque. If there’s no torque, there’s no change in angular momentum.

The angular momentum before I pull the cord in is \( L = I_\omega = (0.25 \text{ kg-m}^2) (5 \text{ radians/second}) = 1.25 \text{ kg-m}^2 / s \).

After I pull in the cord, \( I_a = MR^2 = (0.25 \text{ kg}) (0.5 \text{ m})^2 = \frac{1}{16} \text{ kg-m}^2 \), which is \( \frac{1}{4} I_\omega \).

So the angular velocity after I pull it in has to be \( \omega_a = L/I_a = 20 \text{ radians/second} \), or \( 4 \times \omega_\omega \).

So the final kinetic energy is \[ \frac{1}{2}I_a\omega_a^2 = \frac{1}{2} \left( \frac{1}{16} \text{ kg-m}^2 \right) (20 \text{ rad/s})^2 = 12 \text{ Joules} \], or \( 4 \times \) the initial kinetic energy. That means I did 9 Joules of work pulling in the mass!