Q1). A ball is thrown with an initial velocity \( v_0 \) at an angle \( \theta \) with the vertical. On its way down, the ball gets stuck in a tree at height \( H \). A \( y \) vs \( x \) plot of its motion is given in Fig 1 below.

a). In term of unit vectors \( \hat{i} \) and \( \hat{j} \), write down the initial velocity vector \( \vec{v}_i \).

\[
\vec{v}_i = v_0 \sin \theta \hat{i} + v_0 \cos \theta \hat{j}
\]

b). Write down the velocity vector \( \vec{v}_{top} \) at the instant the ball reaches the peak of its trajectory. Indicate this velocity vector on the plot above.

\[
\vec{v}_{top} = v_0 \sin \theta \hat{i} + 0 \hat{j}
\]

c). Figure out \( \Delta \theta \) between \( t = 0 \) and \( t = t_{max} \) graphically i.e., by using the head-to-tail rule of vector addition. Here, \( t = t_{max} \) is the instant the ball reaches its maximum height. How is the direction of the average acceleration vector related to \( \Delta \theta \)? What about its magnitude?

\[
\Delta \vec{v} = \vec{v}_i - \vec{v}_{top} = V_0 \cos \theta \hat{i} \quad \text{and} \quad \Delta \vec{a} = \frac{\Delta \vec{v}}{\Delta t}
\]

The direction of the average acceleration \( \vec{a} \) is the same as \( \Delta \vec{v} \). The magnitude \( |\Delta \vec{a}| = |\Delta \vec{v}| / \Delta t \).

d). Using \( y(t) = y_0 + v_0 t + \frac{1}{2} a_s t^2 \), set up a quadratic equation to find the time it takes the ball to reach the tree at height \( H \).

\[
y = y_0 + v_0 t + \frac{1}{2} g t^2 = 0
\]

\[
\frac{1}{2} g t^2 - v_0 \cos \theta t + y - y_0 = 0
\]

\[
t^2 - \frac{2 v_0 \cos \theta}{g} t + \frac{2 y - y_0}{g} = 0
\]

e). Look at Fig 1 again carefully. How many valid solutions do you expect to find for \( t \) from this quadratic equation? What do each of these solutions correspond to?

2 Solutions, the first corresponds to \( \dot{y} = 0 \) with \( \frac{dy}{dt} < 0 \) the second \( \dot{y} = H, \frac{dy}{dt} > 0 \).
Q1). A ball is thrown with an initial velocity \( v_0 \) at an angle of \( \theta \) above the horizontal. A \( y \) vs \( x \) plot of its motion is given below.

a). Find the time \( t_{\text{up}} \) for the ball to reach its maximum height \( h_{\text{max}} \) in terms of \( v_0 \), \( \theta \) and \( g \).

\[
\frac{t}{2} = \frac{v_0 \sin \theta}{g}
\]

b). Find the time \( t_{\text{down}} \) for the ball to travel from \( y = h_{\text{max}} \) back down to \( y = 0 \).

What seems to be the relationship between \( t_{\text{up}} \) and \( t_{\text{down}} \)?

\[
y_y = y_0 + v_{y0} t - \frac{1}{2} g t^2
\]

\[
h = 0 + v_{y0} t - \frac{1}{2} g t^2
\]

\[
\frac{t}{2} = \frac{v_0 \sin \theta - g t}{g}
\]

c). Find the total distance \( x_{\text{total}} \) travelled by the ball in the \( x \)-direction in terms of \( v_0 \), \( \theta \) and \( g \).

\[
D = \frac{(v_0 \cos \theta)^2 + (v_0 \sin \theta)^2}{g}
\]

d & e & f). For a fixed \( v_0 \), suppose we increase \( \theta \). Let's call the smaller angle \( \theta_1 \) and the larger angle \( \theta_2 \).

d). Compare the total time of flight \( t_{\text{flight}}^{(1)} \) with \( t_{\text{flight}}^{(2)} \).

\[
\theta_2 > \theta_1 \Rightarrow \frac{\pi}{4} \Rightarrow v_{y2} > v_{y1} \Rightarrow t_{\text{flight}}^{(2)} > t_{\text{flight}}^{(1)}
\]

\[
v_{y2} = v_{y0} - g t_{\text{flight}}^{(2)}
\]

e). What would happen to \( h_{\text{max}} \)? Why?

\[
h_{\text{max}}^{(2)} > h_{\text{max}}^{(1)} \text{ because } v_{y2} > v_{y1}
\]

\[
h = \frac{v_{y2}^2}{2 g} = \frac{v_{y0}^2}{2 g}
\]

f). What would happen to the range \( x_{\text{total}} \)? Is it possible for \( h_{\text{max}} \) to change but \( x_{\text{total}} \) to remain the same? If so, find the condition that must be satisfied between \( \theta_1 \) and \( \theta_2 \) if \( x_{\text{total}}^{(1)} = x_{\text{total}}^{(2)} \).

If \( \theta_2 \theta_1 \), then \( x_{\text{total}}^{(2)} > x_{\text{total}}^{(1)} \).

\[
v_1 = v_0 - \alpha \epsilon \Rightarrow D = \frac{\alpha v_0^2}{2 g} \sin \theta \cos \theta
\]

\[
\frac{t}{2} = \frac{v_0 \sin \theta - \frac{1}{2} g t}{g}
\]

\[
\frac{D_2}{D_1} = \frac{\alpha}{j} = \frac{2 \sin \theta_1 \cos \theta_1}{\sin \theta_2 \cos \theta_2}
\]

\[
\Rightarrow \sin \theta_1 \cos \theta_1 \Rightarrow \sin \theta_2 \cos \theta_2
\]
Q1). A ball is thrown with an initial velocity \( \vec{v}_i \) with magnitude \( v_0 \) at an angle \( \theta \) above the horizontal. A \( y \) vs \( x \) plot of its motion is given below.

a). Find the time \( t_{up} \) for the ball to reach its maximum height \( h_{max} \) in terms of \( v_0 \), \( \theta \) and \( g \).

\[
\begin{align*}
V_{iy} &= V_{0y} + at \\
0 &= V_{0y} \sin \theta - gt \\
t &= \frac{V_{0y} \sin \theta}{g}
\end{align*}
\]

b). Find the time \( t_{down} \) for the ball to travel from \( y = h_{max} \) back down to \( y = 0 \).

What seems to be the relationship between \( t_{up} \) and \( t_{down} \)?

\[
\begin{align*}
y_t &= V_{0y} + V_{0y} + \frac{1}{2}gt^2 \\
h &= V_{0y} + \frac{1}{2}gt^2 \\
0 &= V_{0y} + \frac{1}{2}gt^2 \\
n &= V_{0y} + \frac{1}{2}gt^2 \\
t &= \frac{V_{0y} \sin \theta}{g}
\end{align*}
\]

c). Find the total distance \( x_{total} \) traveled by the ball in the \( x \)-direction in terms of \( v_0 \), \( \theta \) and \( g \).

\[
\begin{align*}
t_{up} &= \frac{V_{0y} \sin \theta}{g} \\
\Rightarrow x_{total} &= V_x t = 2\frac{V_0 \sin \theta \cos \theta}{g}
\end{align*}
\]

d). Find the final vertical velocity \( v_{fy} \) and the final horizontal velocity \( v_{fx} \) of the ball right before it hits the ground. What angle does the final velocity vector make with the horizontal?

\[
\begin{align*}
\theta \quad V_{f, y} &= -V_{i, y} \Rightarrow V_{f, y} = -V_0 \sin \theta \\
\text{\( V_f \) makes an angle of \( \theta \) below the horizontal.}
\end{align*}
\]

e). How does the magnitude of initial velocity \( v_i \) compare with the magnitude of the final velocity \( v_f \)?

\[
\begin{align*}
\vec{V}_i &= V_0 \cos \theta \hat{i} + V_0 \sin \theta \hat{j} \\
\vec{V}_f &= V_0 \cos \theta \hat{i} - V_0 \sin \theta \hat{j} \\
|V_i| &= |V_f| \\
\frac{V_f}{V_i} = \sqrt{\left(\frac{V_0 \cos \theta}{V_0 \cos \theta}\right)^2 + \left(\frac{V_0 \sin \theta}{V_0 \sin \theta}\right)^2}
\end{align*}
\]
f). Mark a pair of points (of your choice) on the trajectory of the ball that the same vertical position \( y \). What implications does your result for part (e) have for the ball's vertical velocity at any such pair of points.

\[
\begin{align*}
|V_i| &= |V_0| \\
V_{fy} &= -V_{iy} \\
V_{x_a} &= V_{x_b}
\end{align*}
\]