SOLUTIONS - 9
CHAPTERS - 20, 21.

**FORMULATE**

Faraday - Lenz's law

\[ E = \mathbf{E}_c \cdot \mathbf{E}_N \cdot \mathbf{c} = -\frac{\Delta \Phi_B}{\Delta t} \]

\[ \Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos(\hat{n}, \mathbf{B}) \]

**INDUCTANCE**

\[ L = -\frac{E}{\Delta t} \quad (L = N \Phi_B) \]

Self Inductance

Solenoid \( L = \mu_0 n^2 A \) \quad \mathbf{B} = \mu_0 n \mathbf{I} \quad \mathbf{m}

\[ \mathbf{U}_B = \frac{1}{2} L \mathbf{I}^2 \]

\[ \eta_B = \frac{\mathbf{B}^2}{2 \mu_0} \]

Generator

\[ E = NBA \omega < \omega B \]

**EM WAVES**

\[ C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \mathbf{I} = \frac{P}{4\pi R^2} \]

\[ I = \frac{1}{2} C \varepsilon_0 \varepsilon_m^2 = \frac{1}{2} \frac{C B_m^2}{\mu_0} = \frac{E_m B_m}{2 \mu_0} \]

\[ S = \frac{E \times B}{\mu_0} \]
20-33

No. of turns \( N \) in the coil \( \Rightarrow N = 10 \).

Major axis \( 2a = 10.0 \text{ cm} = 0.1 \text{ m} \)

Minor axis \( 2b = 4.0 \text{ cm} = 0.04 \text{ m} \)

Speed of rotation \( \dot{\varphi} \) \( \Rightarrow \omega = 100 \text{ rpm} = 100 \text{ rotations per minute} \)

\[ \omega = \frac{10 \pi \text{ rad/s}}{3} \]

Magnitude of Earth's magnetic field \( B \): \( B = 55 \mu \text{T} = 55 \times 10^{-6} \text{ T} \)

(a) What is the maximum voltage induced in the coil if the axis of rotation is along the MAJOR AXIS and aligned PERPENDICULAR to the Earth's magnetic field \( \dot{\psi} \)?

In this case, the flux through the coil keeps changing, reaching a maximum when the plane of the coil is perpendicular to \( B \)-field.
And, the maximum voltage induced is given by:

\[ E_{\text{max}} = NBA \omega \]

\[ A = \pi a b = \pi (2a)(2b) \div 4 \]

\[ E_{\text{max}} = \frac{NBA \omega}{4} \]

\[ = (10) \left( 55 \times 10^{-6} \text{T} \right) \left( \pi \times (0.01)(0.004) \right) m^2 \left( \frac{10 \pi \text{ rad/s}}{3} \right) \]

\[ = \frac{55 \times \pi}{3} \times (0.01)(10)^2(0.004)^2 \times 10^{-6} \text{ V} \]

\[ E_{\text{max}} = 18.1 \times 10^{-6} \text{ V} = 18.1 \mu\text{V} \]

(b). When the axis of rotation is aligned along the Earth's magnetic field.

In this case, the area of the loop perpendicular to the \( B \) field is zero.

So, the flux is zero always.

So, the max. induced voltage \( E_{\text{max}} = 0 \).
Radius of the solenoid = \( r = 2.5 \text{ cm} = 0.025 \text{ m} \).

No. of turns = \( N = 400 \) turns.

Length of the solenoid = \( l = 20 \text{ cm} = 0.2 \text{ m} \).

(a) Inductance:

\[
L = \frac{\mu_0 \cdot N^2 A}{l}
\]

\[
A = \pi r^2 \quad ; \quad \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}
\]

\[
L = \frac{(4\pi \times 10^{-7}) \cdot (400)^2 \cdot (\pi \times (0.025)^2)}{0.2}
\]

\[
= \frac{(4\pi) \cdot (16) \cdot (\pi) \cdot (25)^2 \times 10^{-7} \times 10^4 \times 10^{-6} \times 10^{-2}}{2}
\]

\[
L = 0.002 \text{ H}
\]

\[
L = 2 \text{ mH}
\]

(b) We want to produce an emf of \( \mathcal{E} = 75 \text{ mV} \).

Given \( L = 2 \text{ mH} \), what is \( \frac{\Delta I}{\Delta t} \), the rate of change of \( I \)?

We know:

\[
\mathcal{E} = -L \cdot \frac{\Delta I}{\Delta t} \Rightarrow \left| \frac{\Delta I}{\Delta t} \right| = \frac{\mathcal{E}}{L} = \frac{-75 \text{ mV}}{2 \times 10^{-3} \text{ H}}
\]

\[
\frac{\Delta I}{\Delta t} = 37.5 \text{ A/s} \quad \text{Current changes at 37.5 A/s.}
\]
\[ E = -\frac{\Delta \Phi_B}{\Delta t} \]

The South pole of the magnet is being moved toward the loop.

To answer whether \( V_a - V_b \) is > 0, < 0 and = 0, we first look at the \( B \) field lines around a bar magnet.

As shown in the figure, the flux through the loop in the direction opposite to the direction of motion of the magnet increases as the South pole approaches the loop. This is because magnetic field lines start at the North pole and end on the South pole. This increase in flux, according to Faraday's law, induces a current in the loop. But Lenz's law gives the direction of this current.

Lenz's Law: The direction of the current induced in a loop because of a changing magnetic flux is such that the...
the change in flux is opposed.

In the present case, we apply this law by saying that, as the flux through the loop from \textbf{Right to left} in the figure is increasing, the current induced in the loop opposes this by producing a magnetic flux going \underline{from \textbf{Left to Right}}.

 Flux to be generated \underline{left to right}
 Flux entering \underline{right to left}

But, by the right hand rule, the current has to be as shown below to produce a flux from \textbf{Left to Right}.

So, current flows from \underline{b to a}. But current flows from higher potential to lower.

\( V_b > V_a \Rightarrow V_b - V_a > 0 \) or \( V_a - V_b < 0 \)
\[ f = 2.45 \times 10^9 \text{ Hz} \]
\[ |r - r_1| = 6 \text{ cm} = \frac{1}{2} \]
\[ A = 12 \text{ cm} \]
\[ v = f \lambda = 0.12 \text{ m} \cdot 2.45 \times 10^9 \text{ Hz} = 2.94 \times 10^8 \text{ m/s} \]

1-48
\[ I = 1340 \text{ W/m}^2 \]
\[ P = I \cdot A \]
Area encountering radiation = \( 4\pi r^2 \)
\[ r = 1.49 \times 10^8 \text{ m} \]
\[ P = 1340 \text{ W/m}^2 \cdot \left(1.49 \times 10^8\right)^2 \text{ m}^2 \cdot 4\pi = 3.73 \times 10^{26} \]

1-49
\[ I = 1340 \text{ W/m}^2 \]
\[ I = \frac{1}{2} E_{\text{max}}^2 E_0 \cdot C \]
\[ E_{\text{max}} = \left( \frac{2 \cdot I}{C E_0} \right)^{1/2} = \left( \frac{2 \times 1340}{3 \times 10^8 \times 9 \times 10^{-12}} \right)^{1/2} = 9.96 \times 10^2 \text{ N/C} \]
\[ E_{\text{max}} = 9.96 \times 10^2 \text{ N/C} \]
\[ B = \frac{E}{c} = \frac{9.96 \times 10^2}{3 \times 10^8} = 3.3 \times 10^{-6} \text{ T} \]
1-50

\[ f = 27.33 \text{ MHz} = 27.33 \times 10^6 \text{ Hz} \]

\[ v = c \quad (\text{EM wave}) \]

\[ v = f A \quad \Rightarrow \frac{A}{f} = \frac{v}{c} = \frac{3 \times 10^8 \text{ m/s}}{27.33 \times 10^6 \text{ Hz}} \]

\[ A = 11.3 \text{ m} \]

1-52

\[ v = \frac{x}{t} \quad \text{radio waves} \quad v = c \]

\[ t = \frac{x}{v} = \frac{100 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-4} \text{ s} \]

\[ \text{sound waves} \quad v = 343 \text{ m/s} \]

\[ t = \frac{3.0 \text{ m}}{343 \text{ m/s}} = 8.7 \times 10^{-3} \text{ s} \]

radio listeners receive news first

1-54

\[ f_0 = f_s \left( 1 + \frac{u}{c} \right) \]

\[ f_0 = 6.0 \times 10^{-4} \text{ Hz} \left( 1 + \frac{1.8 \times 10^5 \text{ m/s}}{2.9979 \times 10^8 \text{ m/s}} \right) \]

\[ f_0 = 6.0340 \times 10^{-4} \text{ Hz} \]

note: we get the same value approximating \( c \) as \( 3 \times 10^8 \) m/s. This is an excellent and widely used approx. [1]
(a) \[ E_{\text{max}} = 0.20 \times 10^{-6} \text{ V/m} \]

\[
B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{0.20 \times 10^{-6}}{3 \times 10^8} \text{ m/s} = 6.67 \times 10^{-10} \text{ T}
\]

(b) \[
I = \frac{1}{2} \varepsilon_0 c E_{\text{max}}^2 = \frac{1}{2} \times 9 \times 10^{-12} \times 3 \times 10^8 \times (0.2)^2 \times 10^{-12} = 5.3 \times 10^{-17} \text{ W/m}^2
\]

(c) \[ I = \frac{P}{A} \]

Area of dish = \(\pi (\frac{d}{2})^2 = \pi \cdot 100\)

\[ P = 5.3 \times 10^{-17} \text{ W/m}^2 \times 100 \pi \text{ m}^2 = 1.7 \times 10^{-14} \text{ W} \]

22-2

\[ c = \frac{2d}{t} \]

time for one full revolution = \(\frac{1}{27.5} \text{ s} \)

time between notch and neighboring tooth = \(\frac{t_{\text{rev}}}{2 \cdot \# \text{ teeth}} \)

t = \frac{27.5}{2 \cdot 3600} = 5.65 \times 10^{-5} \]

\[ c = \frac{2.75 \text{ km}}{2.97 \times 10^5 \text{ m/s}} = 9.27 \times 10^{-5} \text{ m/s} \]