Formulas

1. \( \vec{F} = q(\vec{v} \times \vec{B}) \)  
   "Lorentz Force" - force on a moving charge due to a \( \vec{B} \)-field

2. \( \vec{F} = I \vec{l} \times \vec{B} \)  
   force on a current-carrying wire due to \( \vec{B} \)-field

3. \( \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{r} \)  
   magnetic field due to a current
   **unit vector always tangent to concentric circles around wire**
   **direction given by Right Hand Rule (RHR)**
   
   * Thumb in direction of current, fingers curl in same sense as the \( \vec{B} \)-field
   
4. \( \vec{B} = \mu_0 n I \hat{n} \)  
   magnetic field due to a solenoid
   \( \hat{n} \) directed down solenoid axis consistent with RHR
   
   + Fingers curl in direction of current, thumb points parallel to \( \vec{B} \)-field

5. \( R = \frac{m v}{qB} \)  
   cyclotron radius
Total force on the particle is given by (1) + the force due to the electric field.

\[ \vec{F}_1 = q \vec{E} + q (\vec{v} \times \vec{B}) \]

We are looking for situation when \( \vec{F}_1 = 0 \). This occurs if

\[ \vec{E} = -\vec{v} \times \vec{B} \]

\( \vec{v} \) is perpendicular to \( \vec{B} \) and by the RHR we see \( \vec{v} \times \vec{B} \) is directed opposite to \( \vec{E} \). This tells us

\[ \boxed{\vec{E} = \vec{v} \vec{B}} \]

c-condition on \( E, v, \) and \( B \).

\[ \text{19-33} \]

For an elastic collision we know that energy and momentum are conserved.

**Momentum**

\[ M_a \vec{V}_{ai} = M_a \vec{V}_{af} + M_p \vec{V}_{pf} \]

**Energy** (velocities when particles are separated by a large distance)

\[ \frac{1}{2} M_a \vec{V}_{ai}^2 = \frac{1}{2} M_a \vec{V}_{af}^2 + \frac{1}{2} M_p \vec{V}_{pf}^2 \]  
(a)

**momentum components**

\[ \begin{align*}
\text{(x-comp)} & \quad M_a \vec{V}_{ai,x} = M_a \vec{V}_{af,x} + M_p \vec{V}_{pf,x} \\
\text{(y-comp)} & \quad 0 = M_a \vec{V}_{af,y} + M_p \vec{V}_{pf,y}
\end{align*} \]  
(b)

(c)
Because the collision is head-on we ignore components of velocity perpendicular to ray connecting two particles before they collide.

\[ V = \text{initial velocity of } \alpha \]
\[ V_a = \text{final velocity of } \alpha \]
\[ V_p = \text{final velocity of proton} \]

Our equations take a simplified form.

\[ \text{(a) energy: } 2mV^2 = 2mV_a^2 + \frac{1}{2}mV_p^2 \Rightarrow 4V^2 = 4V_a^2 + V_p^2 \]
\[ \text{(b) momentum: } 4mV = 4mV_a + mV_p \Rightarrow 4V = 4V_a + V_p \]

We don't care about finding the exact velocities, only about the relation between \( V_p \) and \( V_a \). We can eliminate \( V \) from these equations.

\[
\begin{align*}
4V_a^2 + V_p^2 &= \frac{1}{4} (4V_a + V_p)^2 \\
4V_a^2 + V_p^2 &= 4V_a^2 + \frac{1}{4}V_p^2 + 2V_a V_p \\
\frac{3}{4}V_p^2 &= 2V_a V_p \\
\frac{3}{4}V_p &= 2V_a
\end{align*}
\]

\[ V_a = \frac{3}{8}V_p \]

We know cyclotron radius of the proton

\[ R = \frac{m_p V_p}{eB} \]

For the \( \alpha \)-particle it will be...
19-37 we know $B$-field from a wire, the magnitude is given by (3).

$$B = \frac{\mu_0 I}{2\pi r}$$

solve for distance: 

$$r = \frac{\mu_0 I}{2\pi B}$$

$I = 20A$ \hspace{1cm} $B = 1.7mT$ \hspace{1cm} $\mu_0 = 4\pi \times 10^{-7} T.m/A$

$$r = \frac{4\pi (20) \times 10^{-7} T.m.A}{2\pi (1.7) \times 10^{-3} A}$$

$$r = \frac{4}{1.7} \times 10^{-3} m = 2.35 \times 10^{-3} m$$

19-41

What is $B$-field at $(x,y)$?

$$B = B_1 + B_2$$

$$B_1(x,y) = \frac{\mu_0 I_1}{2\pi y}$$

$$B_2(x,y) = -\frac{\mu_0 I_2}{2\pi x}$$

$$B(x,y) = \frac{\mu_0}{2\pi} \left( \frac{I_1}{y} - \frac{I_2}{x} \right) \hat{z}$$
\[ \mathbf{B}(x, y) = 2 \times 10^{-7} \left( \frac{9}{4} - \frac{k}{4} \right) \frac{T \cdot m}{A} \]

\[ \mathbf{B} = \frac{2 \left( 28 - 18 \right)}{12} \times 10^{-7} T = 1.66 \times 10^{-7} T \]

**Square loop inside a solenoid.**

\[ \mathbf{B} - \text{field inside a solenoid is uniform.} \]

\[ \mathbf{B} = \mu_0 n I \hat{n} \]

\[ \hat{n} \text{ is perpendicular to plane of loop.} \]

**Force on each side of the loop,**

\[ \mathbf{F}_B = I \mathbf{I} \times \mathbf{B} = I I B \hat{n} \]

**The force on each segment works to pull the loop apart.**

**Its magnitude is**

\[ \mathbf{F}_B = I I B = \mu_0 I I S \hat{n} \]

\[ F_B = 4 \pi \left( 2 \text{cm} \right) \left( 2 \text{A} \right) \left( 15 \text{A} \right) \left( 30 \text{km} \right) \times 10^{-7} \frac{T \cdot m}{A} \]

\[ F_B \approx 9 \times 10^{-3} \text{T} \cdot \text{mA} \approx 9 \times 10^{-3} \text{N} \]
Chapter 20

Formulas

(i) \[ E = \frac{N}{A} \cdot \frac{\Delta \phi_B}{\Delta t} \] "Faraday's law" - change in magnetic flux through a surface with time is equal to the voltage around the boundary of that surface.

(ii) \[ \phi_B = \sum_S \nabla \cdot B \cdot \vec{A} \] magnetic flux through surface \( S \).

(iii) \[ E = N \cdot A \cdot B \cdot \omega \cdot \sin \omega t \] This is the voltage induced around a coil with \( N \) turns, area \( A \), that is rotating with angular velocity \( \omega \). There must be a time changing magnetic flux.

More fundamental form of field \( E \) QN. If flux of \( B \) field varied with time it generates a \textit{non-Coulomb}\,\,\,\,\,E-field in every loop surrounding region where \( \phi_B \) is changing. Hence lines of \( E \)-field circulate around \( \phi_B \). Therefore circulation of \( E \)-field around a closed loop is given by \( -\frac{\Delta \phi_B}{\Delta t} \) \[ \sum (E \cdot d\vec{r}) = -\frac{\Delta \phi_B}{\Delta t} \] (Curl).
The minus sign on \( z \) is crucial as it ensures the sense of \( \mathbf{E} \) is such as to oppose change in \( \mathbf{B} \).

\[
\mathbf{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}
\]

Compute flux of \( \mathbf{B} \) through shaded face.

\[
\Phi_B = \sum \mathbf{B} \cdot \mathbf{A} = \mathbf{B} \cdot \mathbf{A},
\]

\[
\mathbf{A} = \hat{z} \times \mathbf{A} = \hat{z} \times 2 \ell^2
\]

\[
\Phi_B = \ell^2 \hat{z} \times (B_x \hat{x} + B_y \hat{y} + B_z \hat{z}) = \ell^2 \mathbf{B} \mathbf{x}
\]

\[
l = 2.5 \text{ cm} \quad B_x = 5.0 \text{ T}
\]

What is total flux through cube's surface?

\[
\int \Phi_B = 3.125 \times 10^{-3} \text{ Tm}^2
\]

What is total flux through cube's surface?

\[
\int \phi_B = 3.125 \times 10^{-3} \text{ Tm}^2
\]

20-10

Loop is stretched from points A and B until it has zero area. This takes time \( T_0 \), what is the average induced emf around loop?

Use (i)

\[
\Delta \Phi_B = \phi_B, t - 0 = \pi R^2 B
\]

\[
\Delta t = T_0, N = 1
\]

Therefore,

\[
E = -\frac{\pi R^2 B}{T_0} = -\frac{\pi (12 \text{ cm})^2 (0.15 \text{T})}{0.25} \quad \text{V}
\]

\[
E = -\frac{\pi (1.44 \times 0.15)}{2} \times 10^{-3} \text{ Tm}^2 \quad \text{V} = 3.4 \times 10^{-2} \text{ V}
\]

20-17

To find emf we use (i).

Flux through loop

\[
\phi_B = A(t) B_E \cos (28^\circ)
\]

\[
\phi_B = A(t), \mathbf{B}_E
\]
\[ E = -N \frac{\Delta \phi_B}{\Delta t} \]

\[ \Delta \phi_B = (A_f - A_i) B_E \cos(28^0) \]

\[ N = 200, \ B_E = 50.0 \mu T \]

\[ A_f - A_i = 39.0 \text{ cm}^2 \]

\[ \Delta t = 1.8 \text{ s} \]

\[ E = \frac{-200(50) \cos(28^0)}{1.8} 39 \times 10^{-4} \text{ V} \]

\[ E = -21.66 \text{ V} \]

The car should travel East to maximize induced voltage in the desired way. 

\[ \vec{F} = -e (\vec{V} \times \vec{B}) \] is force experienced by electrons in antenna. This reaches a maximum when \( \vec{V} \parallel \vec{B} \Rightarrow \vec{F} = \pm eV \vec{B} \hat{\jmath} \), but we want electrons to move to the bottom of the antenna so

\[ \vec{F} = -eV \vec{B} \hat{\jmath} \] this occurs if \( \vec{V} \) is directed eastward.
the magnitude of the induced emf can be computed 2 ways. First we use Faraday's Law.

\[ \Delta \Phi_B = (\Delta f - A_i) \vec{B} \cdot \vec{E} \cos(65^\circ) \]

\[ \Delta \Phi_B = l \vec{v} \Rightarrow E = l \vec{v} \vec{B} \cdot \vec{E} \cos(65^\circ) \]

OR we can compute force. (MOTIONAL EMF)

\[ \vec{F} = q \vec{v} \times \vec{B} \Rightarrow q \vec{v} \vec{B} \cos(65^\circ) = F \]

\[ F = qE \text{ and } E = \frac{qF}{B} \text{ for a uniform field} \]

\[ E = \frac{(1.2 \text{ m})(65 \text{ km/h})(50 \text{ MT}) \cos(65^\circ)}{3.6} \times 10^3 \times 10^{-6} \times 10^{-6} \cos(65^\circ) = 4.58 \times 10^{-4} \text{ V} \]

20-25

when coil enters field

\[ E = -N \frac{\Delta \Phi_B}{\Delta t} = -NB\vec{v} \vec{w} \]

this will induce a current that opposes flux change (Lenz's Law).

Current will go up.

\[ I = \frac{18}{R} = \frac{NB\vec{v} \vec{w}}{R} \]

this segment is only one that opposes motion.
There will be an opposing force on this coil due to this current interacting with the magnetic field.

\[ F = I \omega B = \frac{N v w^2 B^2}{R} \]

Once the coil is completely in side region with \( B \)-field the force vanishes because there is no longer a change in magnetic flux.

Finally, when coil begin to leave there is an opposing force (Lenz law). The flux begins to decrease which induces a current that interacts with the magnetic field to oppose its motion.

\[ F = \frac{N v w^2 B^2}{R} \]

20-30

From (iii) we know \( f = \frac{1500 \text{rev}}{\text{min}} = \frac{1500}{60} \text{ Hz} \)

\[ E = B N A w \sin \omega t \]

\( \omega = 2\pi \frac{150}{6} \text{ Hz} \)

\[ E = 100 (50 \times 10^{-8} \text{T}) (0.04 \text{ m}^2) 2\pi (150) \frac{1}{6} \sin \omega t \]

\[ E_{\text{max}} = \frac{5 \times 4 \times 2\pi \times 1.5 \times 10^{2+1-6-2+2}}{6} \frac{Tm^2}{s} = \frac{1}{\pi} \times 10^{-2} \text{ V} \]