MAXWELL'S EQUATIONS: RADIATION \Rightarrow \text{LiGHT}

To summarize, the field Equations derived from Experiments are:

**GAUSS' LAW FOR COULOMB** \( E \)

\[
\Sigma_c \frac{E \cdot \Delta A}{\varepsilon_0} = \Sigma Q_i
\]  

**GAUSS' LAW FOR** \( B \)

\[
\Sigma_c B \cdot \Delta A = 0
\]  

**LENZ'S LAW**

\[
\Sigma_c E_{NC} \cdot \Delta l = -\frac{\Delta \Phi_B}{\Delta t}
\]  

**AMPERE'S LAW**

\[
\Sigma_c B \cdot \Delta A = \mu_0 \Sigma I_i
\]

When Maxwell began to study these equations, he realized that there was a serious problem. Scientists believe that at its most fundamental level nature must be symmetric.

Maxwell noticed that whereas a time varying flux of \( B \) gave rise to an \( E \)-field \( E_{NC} \) in Eq. (3) there was no corresponding term in Eq. (4'). He immediately asserted that the above field equations could not be regarded as being complete. This was a FUNDAMENTAL PROBLEM Maxwell also noted a "PRACTICAL PROBLEM" in using Eq. (4'). Imagine that we charge a capacitor to \( \pm q \) and then connect a wire between the two plates as shown.
It is clear that a conduction current \( \frac{\Delta q}{\Delta t} \) begins to flow through the wire and so using Eq. (4') it must create a \( \vec{B} \)-field encircling the wire as shown. However, as soon as you cross one of the capacitor plates, both the current and \( \vec{B} \) must be zero. Again, Maxwell asserted that such a discontinuity cannot be physically meaningful.

To resolve the fundamental problem Maxwell postulated that if the flux of \( \vec{E} \) varies with time it must be equivalent to a current. He called this new type of current a displacement current and introduced the definition

\[
i_D = \varepsilon_0 \frac{\Delta \phi_E}{\Delta t}
\]  

(5)

Of course, Eq. (4') implies that every current generates a \( \vec{B} \) so Maxwell “completed” Eq. (4') by writing

\[
\sum_C \vec{B} \cdot \Delta l = \mu_0 \varepsilon_0 \sum I_C + \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t}
\]

(4)

Where \( I_C \) explicitly signifies a conduction current = flow of charge in a conductor while the second term on the right comes from \( i_D \) [Eq. (5)].

Let us see if introduction of \( i_D \) also solves the practical problem. If the capacitor plates have an area \( A \) the \( \vec{E} \)-field between them is

\[
\vec{E} = -\frac{q}{\varepsilon_0 A} \hat{\vec{x}}, \quad A = A \hat{\vec{x}}
\]

so \( \phi_E = \frac{q}{\varepsilon_0} \)

and \( i_D = \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} = \frac{\Delta q}{\Delta t} = i_C \)

\[ [i_D \text{ is from --ive to +ive because of } \frac{\Delta q}{\Delta t} \text{ is --ive}] \]

Since \( i_D = i_C \) we will have no discontinuity in either the current or the \( \vec{B} \)-field on crossing the capacitor plate.

Maxwell has solved both the fundamental and the practical problem by proposing Eq. (5).
MAXWELL'S EQUATIONS

**Gauss' Law for Coulomb E:**
Since a stationary charge generates a Coulomb E field, the total flux of E Coul through a closed surface is determined solely by the charges located in the volume enclosed by that surface:

\[ \sum \int_{E \text{Coul}} \cdot dA = \frac{1}{\varepsilon_0} \sum q_i \]

(1)

**Gauss' Law for B:**
Since the elementary generators of B are point magnetic dipoles the total flux of B through a closed surface is always zero:

\[ \sum \int_{B} \cdot dA = 0 \]

(2)

**Faraday-Lenz Law**
If the flux of B varies with time a non-Coulomb E field will appear in every closed "loop" surrounding the region where the flux of B is varying. The sense of ENC is invariably such as oppose the variation in the flux of B that causes it; hence, circulation of
The magnetic flux \( \Phi \) around a closed loop is determined by the rate of change of flux of \( B \) through the area within the loop. [Note: Crucial negative sign]:

\[
\sum_{ENc} \cdot \Delta l = -\frac{\Delta \Phi_B}{\Delta t} \quad (3)
\]

**Maxwell- Ampere Law**

Every current generates a \( B \) field that circulates around it. There are two types of current: i) conduction current which involves flow of charge in a conductor and ii) displacement current which arises when flux of \( E \) field varies with time. Hence, circulation of \( B \) around a closed loop is determined by the currents circulating on the surface on which the loop is drawn:

\[
\sum_{C} B \cdot \Delta l = \mu_0 \sum I_c + \mu_0 \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t} \quad (4)
\]
CAUTION: \( i_D \) exists in vacuum. It never involves flow of charge. No conduction current can exist inside the capacitor!!

Maxwell's Equations (1) through (4) have profound consequences. Let us recall his work using these in outer space, where there is vacuum, \( q=0, i_C = 0 \) so the Equations become:

\[
\begin{align*}
\sum_C E \cdot \Delta A &= 0 \quad I \\
\sum_C B \cdot \Delta A &= 0 \quad II \\
\sum_C E_{NC} \cdot \Delta l &= -\frac{\Delta \phi_B}{\Delta t} \quad III \\
\sum_C \mu_0 \varepsilon_0 \Delta l &= \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \quad IV
\end{align*}
\]

and now indeed there is total symmetry with respect to \( E \) and \( B \). This is what led Maxwell to propose that rather than think of \( E \) and \( B \) fields, one should think of a single entity:

\[ \text{Electromagnetic or EM field} \]

And call Equations I through IV, EM field Equations. He next used these Equations to predict that in vacuum there must exist EM-waves! He was able to show that the structure of these Equations is such that both the \( E \) and \( B \) have the functional form (propagation along \( x \) for example) \( f(x \pm ct) \). That is, they propagate as an Electromagnetic wave with the enormous speed \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s} \). This was a giant step forward: Maxwell had solved the problem of the nature of Radiation or Radiant energy. \( \Rightarrow \) Radiation is an Electromagnetic wave. Our observable universe = Matter + Radiation

Incidentally, Einstein demonstrated that matter and radiation convert into one another there by further simplifying our picture of the universe.

→ Heat
→ Light
→ x-rays
→ radiowaves

are all cases of EM waves. They are distinguished only by their frequencies (or wavelengths). We will concentrate on
Finally we come to **light**

Light: is a transverse EM wave (\(E\) and \(B\) fields perpendicular to direction of propagation and also \(E \perp B\)) whose wavelength lies between 400 nm and 800 nm and whose speed in vacuum is \(3 \times 10^8 \text{ m/s}\). As always, light waves transport energy. Let us compare transport of energy by:

**Wave on a string:** Power

\[
P = \frac{1}{2} \mu A^2 \alpha^2 v
\]

\[
v = \sqrt{\frac{F}{\mu}}
\]

**Sound:** Intensity

\[
I = \frac{1}{2} \rho v S_m^2 \omega^2 v
\]

\[
v = \sqrt{\frac{P}{\rho}}
\]

**EM-wave Light:** Intensity

\[
I = \frac{B_m^2}{2 \mu_0} c = \frac{1}{2} \varepsilon_0 E_m^2 c
\]

\[
c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}
\]