DEVICES – AC CIRCUITS

**Battery** Source of Coulomb $E$-field
- Output is emf: $\varepsilon$

**Capacitor** Container for $E$-field $C = \frac{Q}{V}$
- Potential Energy $U_B = \frac{Q^2}{2C}$
- $\eta_E = \text{Energy stored per } m^3 = \frac{1}{2} \varepsilon_0 E^2$
- $\varepsilon_0 = 9 \times 10^{-12} \text{ F/m}$

**Resistor** Represents that it costs energy to transport charge through a conductor

- $V$
- $\rightarrow \text{ MMMM }$
- $R = \frac{V}{I}$
- $J = \sigma E$

**Power loss** $P = I^2 R = \frac{V^2}{R}$

**Inductor** A time varying current causes a Non-Coulomb $E$-field, induced emf, $L = \frac{-\varepsilon}{\Delta i/\Delta t}$

- Container for $B$-field, Potential Energy $U_B = \frac{1}{2} Li^2$
- $\eta_B = \text{Energy stored per } m^3 = \frac{B^2}{2\mu_0}$

**A.C. Generator** Wire loops of area $A$ rotated at $\omega \text{ rad/s}$ in a $B$-field. Generates non-coulomb $E$-field in the loops, produces an emf: $\varepsilon = \omega NBA \sin(\omega t)$

Where $N$ = # of turns in the loop. Hence ac-generator

- $\Phi_B = NBA \cos(\omega t)$
  - where $\theta$ is angle
  - between $\hat{A}$ and $\hat{B}$
  - Rottn by $\omega \text{ rad/s}$ makes $\omega = \omega t$, $\Phi_B = NBA \cos(\omega t)$
  - So $\frac{\Delta \Phi_B}{\Delta t} = -NBIA \omega \sin(\omega t)$, $\varepsilon = -\frac{\Delta \Phi_B}{\Delta t} = \omega NBA \sin(\omega t)$
I. RC with battery, close switch at $t=0$, current flows immediately, potential across $C$ appears later $\varepsilon = \frac{q}{C} + iR$

\[ i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} \]

\[ v_c = \varepsilon \left[ 1 - e^{-\frac{t}{RC}} \right] \]

\[ \tau = RC \]

III. LC-Circuit: Undamped Oscillator

First charge $C$ to $Q_0$. Close switch at $t=0$. Energy stored in capacitor $U_c = \frac{Q_0^2}{2C}$

Charge begins to flow. Total Energy $= (\text{Energy in } E\text{-field}) + (\text{Energy in } B\text{-field})$

$= (\text{Energy in } C) + (\text{Energy in } L)$

\[ \frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2} L \left( \frac{\Delta q}{\Delta t} \right)^2 \]
Recognize, similarity to spring-mass oscillator

\[ \frac{1}{2} kA^2 = \frac{1}{2} kx^2 + \frac{1}{2} m \left( \frac{\Delta x}{\Delta t} \right)^2 \]

\[ x \rightarrow q \quad x = A \cos \omega t \]

\[ m \rightarrow L \]

\[ k \rightarrow \frac{1}{C} \]

\[ \omega_0 = \sqrt{\frac{k}{m}} \]

Now

\[ q = Q_0 \cos \omega_0 t \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ E \text{-field collapses giving rise to } B \text{-field and vice versa.} \]

\[ \text{Charge} \]

\[ q = Q_0 \cos \omega_0 t \]

\[ i = -Q_0 \omega_0 \sin \omega_0 t \]

\[ \text{Period } T = \frac{2\pi}{\omega_0} \]
IV. LCR-CIRCUIT: DAMPED OSCILLATOR.

At $t=0$, charge $C$ to $Q_0$ close switch. Now driving $i$ through $R$ costs $i^2R$ per second.

So $\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right)$ is not constant

$$\frac{\Delta \left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right)}{\Delta t} = -i^2R$$

-negative sign on right because energy is being lost ($R$ is getting warmer).

Now $q = Q_0 e^{-\frac{R}{2L}t} \cos \omega t$

$$\omega = \omega_0 \left[1 - \frac{1}{(2\omega_0\tau)^2}\right]^{\frac{1}{2}}$$

$$\tau = \frac{L}{R}$$

$$\omega_0 \tau = \text{Quality factor} = Q_e$$

Note 1: smaller $R$, larger the duration for which the oscillations persist.

Note 2: $R$ plays role of friction; as always energy lost goes to raise temperature. Electrical Equivalent of Heat.
CIRCUITS: AC

I. Resistor and Generator

\[ Y_R = IR \]

so

If \( \varepsilon = \epsilon_0 \sin \omega t \)

\[ i = \frac{\epsilon_0}{R} \sin \omega t \]

Current and voltage are in phase.

Power

\[ P(t) = iv \]

\[ = \frac{\epsilon_0}{R} \sin^2 \omega t \]

\[ \langle \sin^2 \omega t \rangle = \frac{1}{2} \]

averaged over a cycle. \( \langle P \rangle = \frac{\epsilon_0^2}{2R} = \frac{i_0^2 \epsilon_0}{2} = \frac{i_0^2 R}{2} \) and the power loss is as if \( R \) was connected to a

whole battery \( \varepsilon = \frac{\epsilon_0}{\sqrt{2}} \). In this sense one talks of \( \frac{\epsilon_0}{\sqrt{2}} \) and \( \frac{i_0}{\sqrt{2}} \) as root-mean-square or r.m.s.

voltage and current.

RMS voltage in our house, \( V_{rms} = 115 \text{ Volts} \)

\[ f = 60 \text{ Hz} \]

\[ \omega = 377 \text{ rad/sec} \]

\[ V_{\text{max}} = 155 \text{ Volts} \]