1. (30 points) In the figure below are shown two settings of positive and negative charges. The setting on the left has two positive charges; the setting on the right has one negative charges and one positive charge. Each of the charges has the same magnitude.

a) In each setting two positions are labeled. If a test charge were to measure the field at each position, rank the magnitude of the electric field the charge would measure. (15 pts)

You get 4 pts. for each of the relationships (e.g D > C) and 3 pts for recognizing that B = 0. To see how each of these comes about, we draw a free body diagram for a test charge at each pt. When the distance from the test charge to one of the source charges is one unit, take the force to be F. When the charge is at B or D, the test charge is closer to each source charge than it was when at the corner of the square, so the force is feels is bigger than F. (Actually, since the distance of the test charge from the source charge is half the diagonal of a unit square or $\sqrt{2}/2$ and since the electric force goes as the square of the distance, the square of the distance for the middle ones is $2/4 = 1/2$ as big as for the ones at the corner. This means the force between the source charges and a test charge at B or D will be 2F. But you don’t really need to calculate that to get the ordering.) We have to get the directions right, adding as vectors.

D > C = A > B = 0

b) If a test charge of magnitude twice as large as the original test charge were placed at point A, how would the force it feels compare to the force felt by the original test charge when it was placed at that point? (5 pts)

a. same
b. twice as big
c. half as big
d. other

The answer is b. The force a charge feels is proportional to the amount of charge it has be Coulomb’s law.
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2. (25 points) In the figure below is shown a snapshot of a piece of an elastic spring on which a pulse is traveling. Each square of the grid represents 1 cm. In a video of the motion of the pulse, the pulse is observed to move a distance of 2 cm in 0.04 seconds.

a) Three pieces of tape are attached to the spring. They are marked by small circles in the figure and are labeled A, B, and C. At the instant shown, find the velocity (not just the speed!) of each of these bits of tape. (15 pts)

b) If the spring is 2 m long and has a mass of 800 grams, find the tension in the spring. (10 pts)

The velocity is given by \( \langle \dot{v} \rangle = \frac{\Delta x}{\Delta t} \). The velocity at an instant is given by making the time interval very small. As the pulse moves, the bits of the spring do not move to the left or right (in the x direction) but only up or down. As a result, we can deal only with the y velocity, \( \langle v_y \rangle = \frac{\Delta y}{\Delta t} \).

The pulse is travelling with a speed

\[
v_0 = \frac{\Delta x}{\Delta t} = \frac{2 \text{ cm}}{0.04 \text{ s}} = 0.02 \text{ m} = 0.5 \text{ m/s}.
\]

In the time it takes the pulse to move one box (0.02 s) the bit of tape at A will move down one box and the bit of tape at B will not move at all. So we get

\[
v_A = -1 \text{ cm/s} = -0.5 \text{ m/s}
\]

or 50 cm/s downward. (4 pts = 2 for magnitude, 1 for direction, 1 for explanation).

B is easy: velocity = 0. (4 pts = 2 for answer, 2 for explanation). For C we have to be more careful. We can’t let the pulse go for a full box since the tape would move up and then stay at the top for half the time. Half of the time the pulse is moving one box, the tape is moving with the speed we are interested in, but for half the time it is stationary. Since our formula only gives us average velocity, this isn’t going to work. So let’s only let the pulse move for 1/2 box. This takes 0.01 s. In this time interval, the tape moves up 1 box. So we get

\[
v_C = \frac{1 \text{ cm}}{0.01 \text{ s}} = 100 \text{ cm/s}
\]

The answer is 100 cm/s upward. (4 pts = 2 for magnitude, 1 for direction, 1 for explanation.) If you didn’t calculate the speed of the pulse, the explanation points were increased by 1 each.

b) If the spring is 2 m long and has a mass of 800 grams, find the tension in the spring. (10 pts)

The speed of a pulse on a spring is given by

\[
v_0 = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{T L}{m}}
\]

where \( T \) is the tension and \( \mu \) is the mass density, \( m/L \). If you don’t remember this, it’s
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3. (15 points)  A fluorescent bulb delivers the same amount of light using much less power. If one kW-hr costs 5¢, estimate the amount of money you would save each month by replacing all the 75 W incandescent bulbs in your house by 10 W fluorescent ones. Be sure to clearly state your assumptions and how you came to the numbers you estimated, since grading on this problem will be mostly based on your reasoning, not on your answer.

The problem requires calculation of the savings per month. To do this, we need to consider the amount of energy used by the bulbs and the cost per kilowatt-hour (kWh).

- **Cost per kWh**: $0.05
- **Energy used by incandescent bulbs**: 15 kWh per month
- **Energy used by fluorescent bulbs**: 4.5 kWh per month
- **Savings per month**: $0.90

4. (10 points) In lecture and in homework, we considered the pressure in a water pipe as an analogy for electric potential. Discuss the extent to which this is a good analogy, mentioning at least one characteristic of the water flow analogy that has a parallel in electric current and one feature of electric current that does not have a good analog in the water flow analogy.

Water pressure is a pretty good analogy for electric potential. In a “resistive element” in a water flow system (a pipe with significant drag), the pressure has to drop in order to provide a force ($F = (P_L - P_R)A = \Delta P A$) balancing the drag to keep the water flowing at a constant rate. In the same way, the voltage drops across a resistor to provide an electric force ($F = qE = q\Delta V/L$) balancing the drag on the electrons in the resistor. We even got identical equations:

\[ \Delta P = IR \quad \text{(for water in a pipe)} \]
\[ \Delta V = IR \quad \text{(for electric current in a resistor)} \]

This makes a total of 9 bulbs on for 7 hours/day and 9 bulbs on for 17 hours/day.

The appropriate thing for me to calculate is Watt-hours, since “Watt” is a rate of delivery of energy (energy/time), “Watt-hour” is an energy and that’s what I pay for (in kWh = 1000 Watt-hours).

- **Total energy used by incandescent bulbs**: 11475 W-h
- **Total energy used by fluorescent bulbs**: 4725 W-h
- **Energy saved**: 6750 W-h

This would cost me $337.50 or about $30 per month.

Using 10 W fluorescents would reduce the cost by a factor of 10/75 saving me 65/75 of the cost or $22.50. I would save almost $20/month by using all fluorescents. (This density has to be weighed against the extra cost of the bulbs and their various lifetimes before you decide whether it’s worth switching to all fluorescents.)
voltage is not associated with a force. You only get a force (from an E field) when the voltage changes \( (E = -\Delta V/\Delta x) \).

(Points were assigned for a variety of answers. Basically, you were given 5 pts for discussion of the analogy and what good it did and 5 pts for a limitation. If you discussed the value of the water analogy in general, and in particular for current conservation, you were given 2. If you discussed what you didn’t understand about the analogy you were given 2, even if the analogy actually works where you said it didn’t. Clarity and coherence of expression mattered here in getting your full credit.)

5. (25 points) An electrical circuit is connected as shown in the figure on the right. It contains two identical batteries (labeled #1 and #2), each rated at 6 Volts, and 3 identical bulbs (labeled A, B, and C). At the currents in the circuit, each bulb has almost a constant resistance of 3 Ohms.

A. Would you describe bulb A as being in series with bulb B? in parallel with bulb B? or neither. Explain why you made the choice you did. (10 pts)

Two elements are in series means that whatever current flows through one element must be same as the current that flows through the other. The emphasis here is on “must.” That is, the current equality results from the geometry of the situation — there is no split between them — not from a simple equality of values. (Two elements in parallel could have the same value of the current; that doesn’t make them in series.) Therefore A and B are not in series. (4 pts for answer, 1 pt for reason)

Two elements are in parallel means that whatever voltage drop there is across one, the other must experience the same voltage drop. Again, this must be because of geometry, not just equality of values. Basically, this means their top ends have to be connected together and their bottom ends connected directly together — with nothing in between. This isn’t the case here. Although the top ends of A and B are connected with nothing in between, their bottom ends have a battery in the way. The answer is neither. (4 pts for answer, 1 pt for reason)

B. Rank the brightness of the three bulbs, explaining your choice of ranking. (10 pts)

\[ B > A = C \]

A and C are in identical situations. The rest of the circuit looks the same to each of them. Therefore they have to have the same currents. Call it I. At the upper node in the middle you have currents of I coming in each way so there must be 2I going out the bottom into bulb B. Bulb B therefore has twice the current of A or C. (4 pts for B>A or C, 4 pts for A=C, 2 pts for reasoning.)

C. Find the current in battery #1 if bulb B is removed from its socket. (5 pts)

If B is removed, we have a simple single loop with two resistors and two batteries. But the batteries are reversed. Using the loop rule, one battery produces a rise, the other an equal drop. The result is 0 so there is no overall rise to push a current. There is no current. (3 pts for answer, 2 for reason. If you didn’t notice the batteries were reversed and calculated 12 Volts / 6 Ohms = 2 Amps you got 2 pts.)

D. Find the current in battery #1 if bulb B is present (in the original configuration). (5 pts)

We solve this by using Kirchoff’s principles. First, let’s label the current through battery #1 as I. Since the situation is symmetric, the current through battery #2 must also be I. When they get to the node at the top above B, they have no place to go except down through B. By K1, the current through B is therefore 2I. (At the bottom it splits again, 1/2 going to #1 and 1/2 going to #2.)

We then use K2, the drops = rises around any loop rule. Using the loop shown by the dashed line we get

\[ \sum V_{\text{rises}} = \sum V_{\text{drops}} \]

\[ V_{\text{r1}} = \Delta V_A + \Delta V_B \]

Using K3 (Ohm’s law) with the appropriate currents gives

\[ V_{\text{r1}} = IR + (2I)R = 3IR \]

\[ I = \frac{V_{\text{r1}}}{3R} = \frac{6 \text{ Volts}}{3 (3 \text{ Ohms})} = \frac{2}{3} \text{ Amp} \]