1. (21 points) For the next three problems choose the best answer and write it in the box to the right of the problem. If more than one answer is equally the best, give them all. If no answers are correct, write N. For this problem, neglect gravity, friction, and air resistance.

A positive charge is placed at rest at the center of a region of space in which there is a uniform, electric field. (A uniform field is one whose strength and direction are the same at all points within the region.)

a) When the positive charge is released from rest in the uniform electric field, what will its subsequent motion be?
   (a) It will move at a constant speed.
   (b) It will move at a constant velocity.
   (c) It will move at a constant acceleration.
   (d) It will move with a linearly changing acceleration.
   (e) It will remain at rest in its initial position.

This is because in a uniform electric field the particle always feels the same force, $F = qE$. If a particle feels a constant force, by N2, it has a constant acceleration.

b) What happens to the electric potential energy of the positive charge, after the charge is released from rest in the uniform electric field?
   (a) It will remain constant because the electric field is uniform.
   (b) It will remain constant because the charge remains at rest.
   (c) It will increase because the charge will move in the direction of the electric field.
   (d) It will decrease because the charge will move in the direction opposite to the electric field.
   (e) It will decrease because the charge will move in the direction of the electric field.

This is just like a body falling in a constant gravitational field. The object moves in the direction of the field (down) and because of that the PE drops.

c) A positive charge might be placed at one of two different locations in a region where there is a uniform electric field, as shown below.

2. (24 points) For the following problems select the best answer and write it on the line at the right of the problem. If more than one answer is equally the best, give them all. If no answers work, write N.

In the figure below are shown four one-dimensional arrangements of charge. Each of the charges have the same magnitude, but some are positive and some are negative. In each diagram a point is labeled “P”.

i) In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the largest? (6 pts) ___B_____

Here, you have to pay attention to both the sign of the source charge and how far away it is. If a charge in the next place to the P produces a force of magnitude F, a force two spaces away will produce a force $F/4$ since double the distance between charges reduces their forces by a factor of 4. Taking a force

*** Good Luck ***
to the right, in A a test charge at point P would feel a force +F (from the closest charge) and a force +F/4 (from the charge twice as far away) and a force +F/9 (from the charge three times as far away). B is bigger because both the neighboring + charge and – charge produce a force +F in the positive direction. Nothing else comes as close.

ii) In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the smallest? (6 pts) __________ C

C is the smallest because the two + charges on either side have a canceling effect, leaving only a result due to the third charge which is twice as far away. The total force for this case is +F/4.

In the figure below are shown four two-dimensional arrangements of charge. Each of the charges have the same magnitude, but some are positive and some are negative. In each diagram a point is labeled “P”.

iii) In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the largest? (6 pts) __________ C, D

iv) In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the smallest? (6 pts) __________ B

In these case the angle between the forces matters as much as the distance. In cases A and B, the lower charge is a bit farther away (by a factor of √2 so the force from those is half as big in magnitude as the ones from the straight over or straight down charges). The forces produced by each charge at P is shown in the figure below.

When the two vectors are added in cases C and D you get vectors of the same length. Those in A and B are shorter, with B being the shortest resultant since the two vectors in that case are pushing more in the opposite directions than are the vectors in A which are working more together.

3. (15 points) In a physics experiment to measure the speed of light, two parallel metal plates are charged as a capacitor as shown in the figure. The voltage difference between the plates is set at 100 Volts. The plates have a radius of 10 cm and the distance between them is 0.5 cm. Estimate the magnitude of the electric field at a point between the two plates and the amount of charge on each plate.

Be sure to clearly state your assumptions, since grading on this problem will be mostly based on your reasoning, not on your answer.

One or more of the following numbers might be useful in solving this problem:

<table>
<thead>
<tr>
<th>Number</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.718</td>
<td>9 x 10^9 N-m^2/C^2</td>
</tr>
<tr>
<td>1.6 x 10^{-19} C</td>
<td></td>
</tr>
<tr>
<td>3.1415</td>
<td>0.667 x 10^{-10} N-m^2/kg^2</td>
</tr>
<tr>
<td>0.667 x 10^{-10} N-m^2/kg^2</td>
<td>9.1 x 10^{-31} kg</td>
</tr>
<tr>
<td>3 x 10^8 m/s</td>
<td></td>
</tr>
</tbody>
</table>

We can get the electric field from knowing the relation between the E field and the potential, ∆V = - E ∆x. (3 pts) Since we don’t care about the direction we can ignore the signs. This gives

\[ E = \frac{\Delta V}{\Delta x} = \frac{(100 \text{ V})}{(0.005 \text{ m})} = 0.2 \times 10^5 \text{ V/m} \]

To get the charge, since the plates are flat and close so we can basically treat them like infinite plates in order to relate the field to the charge. (2 pts) This would give us

\[ E = 4\pi\kappa \sigma \]

where \( \sigma \) is the charge density, Q/A. (2 pts) We know E now and we recognize that \( k = 9 \times 10^9 \text{ N-m}^2/\text{C}^2 \), (1 pt) so we can find \( \sigma \). From this, we can calculate A (the area of the plates) and calculate Q. (3 pts)

\[ \sigma = E/4\pi k \]

\[ Q = \sigma A = E A/4\pi k = E(\pi R^2)/4\pi k = ER^2/4k \]

\[ Q = (0.2 \times 10^5 \text{ N/C})(0.1 \text{ m})^2/4(9 \times 10^9 \text{ N-m}^2/\text{C}^2) \approx 0.5 \times 10^{-8} \text{ C} \]

4. (10 points) In discussing electric current flow, we specified three principles known as Kirchoff’s laws that help us calculate what current flows there will be in any circuit. These principles refer directly to the devices of the circuit (resistances, batteries, etc.) and do not make any reference to our model of
moving charge within an electrical circuit. Pick one of these laws. State it clearly and describe how it follows from the model we developed of wires and resistors made of of two kinds of lots of small electric charges, one set moving and one set stationary.

You can choose any one of the three laws. They are:

K1: Around any loop in a circuit, the sum of the potential rises is equal to the sum of the potential drops.

K2: At any junction (or volume) in a circuit, the amount of electric current flowing into that junction (or volume) is equal to the amount flowing out. (This principle is not true if your volume includes half of a capacitor.)

K3: The current flowing through a resistance, $R$, is determined by the potential drop across that resistance according to $I = \Delta V / R$.

Our model of what is going on in a circuit is that matter consists of two kinds of charges, positive and negative with the overall matter being neutral. The positive charges are heavy and stay fixed, while the negative charges can move freely through the lattice of positive charges. Electric fields lead to forces on the charges and the negative ones move producing a flow (current) in response to those forces.

Each Kirchhoff law follows from this model.

K1: Electric forces, like gravity, leads to a potential energy. For flat-earth gravity, the potential energy is just proportional to the height. If you go around a loop, you must come back to the same height you started with. In a circuit, if you carry any charge around a loop, the net change in its electric potential energy must be zero if you come back to your starting point.

K2: The second law is just a statement that the total amount of negative charge we have is conserved. You can’t have a build-up of charge anywhere in the circuit because you would build up a large repulsion. As a result, when a current is flowing, if no charge is to build up, the same amount that goes in must come out.

K3: The third law (Ohm’s law) followed in our model from the assumption that as the charges moved through a resistor there was a drag on them from the positive charges that was proportional to the velocity. As a result, the charges quickly reached terminal velocity in a resistor in which the electric force balanced the drag force. This balance ($qE = bv$) could be shown to lead to Ohm’s law.

You only had to do one. You got 4 pts for the correct statement of the law, 2 pts for showing a clear understanding of the model, and 4 pts for a correct discussion of how the law followed from the assumptions of the model.

5. (30 points) A set of batteries and bulbs are connected as shown in the figure at the right. All of the bulbs have resistance $R$ and the current through bulb C is $I$ as indicated.

(a) What is the current through bulbs A and B and through the battery? Show your work. (15 pts)

Since we don’t know the potential drop across the battery, the easiest way to do this is to think about the currents.

The current in B is I. (2pts, reasoning 3)

The current through B and the current through C run together at the bottom of the figure, go through the battery and through A before splitting into B and C. Therefore (by K2)

The current in A and in the battery is 2I. (each 2pts, reasoning 3)

(b) What is the EMF provided by the battery? Express your answer in terms of the symbols given ($I$ and $R$) and explain your reasoning briefly. (15 pts)

Recall that “EMF” is just another word for “potential difference”. Therefore, we need to figure out the PD across the battery. Using K1, we can see that the rise in the battery must equal the sum of the PD in A plus the PD in B (or C depending on which loop you take). This is $(2I)R + IR = 3IR$. 